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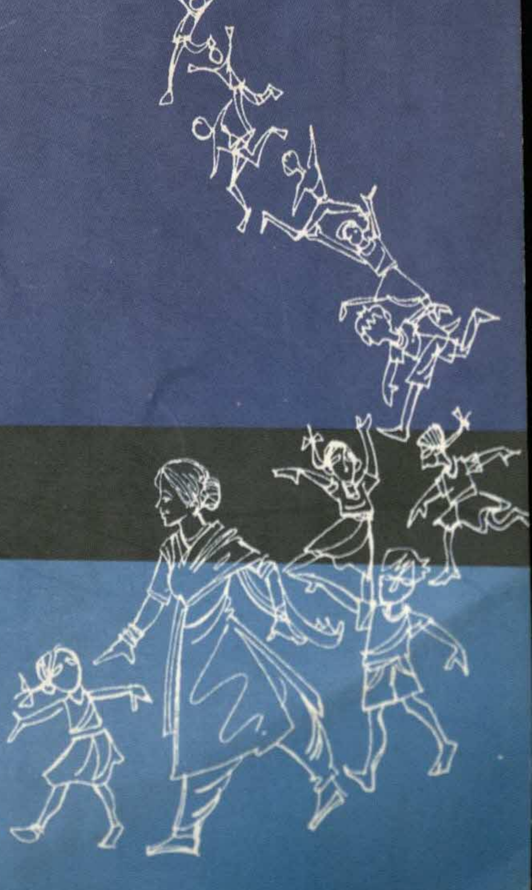
NCERT

Source Book on Assessment for Classes I - V



Mathematics

NCERT

Source Book on Assessment for Classes I-V

MATHEMATICS

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FOREWORD

Following the approval of the *National Curriculum Framework 2005*, NCERT has published innovative textbooks which are based on radically reconceptualised syllabi for different subject areas. The new approach attempts to infuse subject knowledge with the awareness of children's own learning strategies. While reforms in teacher training are being initiated as a parallel process, the challenge of changing the traditional system of examination and evaluation has emerged as a major focus. It was felt that the earlier attempts to define minimum levels of learning need to be revisited, with a view to developing a holistic assessment policy. Such a policy must take into account individual differences and the impact which socio-cultural constraints make on children's ability to make sense of knowledge in different subject areas.

This Source Book signifies NCERT's resolve to provide to teachers and administrators a new vision and approach for assessing children's progress in a system which is accustomed to classifying and labelling children on the basis of a test or examination. Such a system makes it difficult for teachers to perceive each child's progress as an individual trajectory. It also discourages the teacher from recognising the important role that a cooperative classroom culture plays in recognising learning. We hope that this Source Book will encourage such a recognition and will also provide a means to improve record-keeping procedures in primary classes.

We earnestly hope that the Source Book will receive attention in all segments of our complex system of education in India, both in its Centrally-run components, such as Kendriya Vidyalayas, and State-controlled primary schools of different types. The vision of a universalised system of education implies patience and kindness for every child, irrespective of her or his style and pace of learning. Only a system which is kind and gentle to every child can ensure that national progress receives inputs from every citizen.

The development of this Source Book was an arduous collaborative exercise. We wish to thank all the experts and institutions which participated in this exercise at different stages. With assistance from UNESCO, NCERT organised an International Workshop on Assessment. With the cooperation of UNICEF, an earlier draft of this Source Book was tried out in ten States in the country. We wish to thank the regional office of UNICEF, the State Project Directors of SSA, representatives of SCERT's and the district level teams which made the trial of the draft Source Book a successful research endeavour. With the completion of this ambitious project. NCERT's Department of Elementary Education (DEE), especially its Head, Professor K.K. Vashishtha, Professor Sandhya Paranjpe and other colleagues have accomplished a major breakthrough in the long and painful history of evaluation reforms. This project received vital inputs from several other departments of NCERT and its Regional Institutes.

The vision of NCF-2005 emanates from the great struggle symbolised by Gandhi and Tagore to soften the heart of educators towards the little child. This Source Book is dedicated to that struggle.

Krishna Kumar
Director
National Council of Educational
Research and Training

New Delhi
15 September 2008

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This Source Book on Assessment for Classes I-V, a collaborative academic exercise with UNESCO and UNICEF as our partners, owes its present shape and form to the plethora of ideas generated through a series of intensive deliberations in workshops of the different sub-groups constituted for the same. The development process has undergone a number of phases in which experts in the field of education and evaluation, as well as from other disciplines, teachers, representatives of voluntary organisations, NCERT faculty and several other stakeholders participated and made valuable contributions. Significant inputs were also provided by the Core Group Members, State Education Secretaries and other educational functionaries, State Project Offices for SSA in different States and State Councils of Educational Research and Training (SCERTs) that further enriched preparation of the draft document. The consistent support and encouragement from the Elementary Education Bureau of the Ministry of Human Resource Development, Government of India, under the dynamic leadership of Shri Champak Chatterjee (Former Secretary) and Shri A.K. Rath, Secretary (School Education and Literacy) in general and from Ms. Vrinda Sarup Joint Secretary (EE II) in particular is deeply appreciated.

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The draft document has benefited immensely from the large scale field trial conducted in a total number of 160 selected schools across 10 States of the country, namely Assam, Mizoram, Jharkhand, Orissa, Himachal Pradesh, Rajasthan, Tamil Nadu, Kerala, Goa and Maharashtra. The support and co-operation provided by all the State Project Offices and State Councils of Educational Research and Training in these States during the trial period is appreciated. A heartfelt thanks is extended to all the teachers and children in the schools and their parents for participating and providing rich inputs while interacting with the team members.

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The list is by no means exhaustive, and we are grateful to all those who contributed in the making of the Source Books.

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Do You Know?

According to the 86th Constitutional Amendment Act, 2002, free and compulsory education for all children in the 6–14-year age group is now a Fundamental Right under Article 21-A of the Constitution.

EDUCATION IS NEITHER A PRIVILEGE NOR FAVOUR BUT A BASIC HUMAN RIGHT TO WHICH ALL GIRLS AND WOMEN ARE ENTITLED

Give Girls Their Chance !

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1

Introduction

All of us care about children and therefore we are concerned that every school should be a place where each child learns. Of all those involved in children's education, it is the teachers who feel the most responsible. This is expressed in their desire to help all children acquire knowledge, skills, positive attitudes and values and face life with confidence. In order to find out how children are doing in schools, teachers spend a lot of time in assessing children. Most teachers view assessment as an important part in their daily school routine and thus this is a very important part of what they do on a daily basis. Why is this so?

Teachers give a number of reasons for this. One important reason is that it is necessary to know if the child has learnt what she/he was expected to have learnt. The second is to find out what is the child's progress over a certain period of time. However, there is a third reason that has been given more attention by not only teachers but all of us as well i.e. finding out what the child has achieved in different subject areas. This could be because we are all concerned about providing *'good quality'* education and feel that one way of making sure this is happening is by evaluating the child's achievement in the subjects being taught through tests or exams. Testing has its own purpose but if we really want to help children learn better, we need to consider what do marks or grades obtained by children through tests/exams actually tell us about a child's learning or progress.

Assessment – For What?

Let us look at a situation which you as a teacher may have seen or experienced in your school quite often.

In a primary school, 30 children in Class IV were given a test on Environmental Studies, on water a chapter in their textbooks. Varying marks were obtained by the children with most scoring 6 out of 10. Two children, Maithali and Raman scored 8 and 3 out of 10, respectively. When the teacher read out the marks the, children laughed at Raman and made fun of him because he had got low marks. After that day, Raman did not want to come to school at all, and it was difficult for his parents to convince him to go to school.

What do these marks really tell the teacher, parents or others involved in Maithali's or Raman's education? Does it tell anything about what and how both children have learnt and what each of them is capable of doing? Does it tell the teacher how to improve the teaching-learning processes for Maithali and Raman based on their needs? Does it tell the two children, Maithali and Raman anything about their own learning so that they can improve further? Do the marks scored by the children provide a useful report or feedback on what either of the two children know, their progress or learning to their parents or community members?

Many a times, what does happen is that this type of evaluation causes feelings of insecurity, stress, anxiety and humiliation in some children, as in Raman's case, since it brings out what the child does not know rather than what the child knows or can do. It also focusses on assessing the content taught in textbooks and what knowledge children have acquired by rote memorisation. Most of the time, it leads to a comparison among children and creates undesirable competition, even for half a mark. If we, as teachers, want every child to learn then we need to ask ourselves what is it that we are looking for when assessing children?

You will accept the fact that situations like the one presented do come up in schools quite often. These situations lead to questions such as — What should we really be assessing? Are there other ways of assessing children besides tests/exams? Is reporting in terms of marks/ grades sufficient? How should the collected information during assessment help? How can information be collected about children's learning without making teacher's work more difficult? The last question is critical since teachers across the country do face a lot of problems on a daily basis such as dealing with large classes, teaching two, three or even more classes together, and also overcrowded classrooms along with little or no facilities being available in schools. In addition, they have to handle children coming from different backgrounds, speaking different languages and having other special needs. All this requires more time, patience and understanding on the part of teachers and all of us who want to help children realise their full potential. What can be done to help teachers in such situations is a matter of concern that needs to be addressed.

Why the Source Book?

The *Source Book* attempts to answer the questions 'mentioned above' that teachers frequently ask. It hopes to make it easier for teachers to assess children continuously in a comprehensive manner as part of the on-going teaching-learning process, by providing ideas and a variety of options for all teachers to make decisions related to various aspects of assessment. What are those aspects?

The major aspects are:

- Why should children be assessed?
- What should be assessed?
- When should assessment be done?
- How should assessment be done?
- How can assessment information be used?

We hope that there is something for each teacher in this chapter and in the following chapters on Mathematics that will facilitate working in the school and interacting with children more enjoyable and a happy experience for all those involved.

As the *Source Book* is read and the ideas presented in the chapters that follow internalised you may find other ways of understanding *what* and *how* children have learnt and what you can do to make them learn better and enjoy their stay in schools more. To be able to do all this, changes are required in the way assessment is presently being undertaken in classrooms.

Before looking at how assessment can become a part of the ongoing teaching-learning process in terms of the major aspects stated above, it would help if some time is spent on thinking about and answering three important questions that will facilitate the same:

- What should primary education develop in children?
- What kind of a child would you like to see at the end of the primary stage?
- What would the profile of a child be in relation to different subject areas and total personality development?

> Education is ... to educate your children to understand the whole of life and not merely segment of life like the physical, emotional, mental, psychological or spiritual; to have not the compartmental, divided outlook but a whole total integrated outlook on life..." to bring out through education a human being who is creative, who is capable, who possesses that intelligence which is not burdened and which is not shaped in any particular direction but is total, who is not belonging to any particular society, caste or religion so that through that education and with that intelligence he arrives at a maturity and, therefore, is capable of making his life, not merely as a technician but as a human being".
>
> — J. Krishnamurti

Reflection on these three questions will provide the necessary direction to guide the process of assessment that is to be undertaken. As can be seen, these questions focus on the child who is central to teaching-learning that goes on in classrooms.

Understanding Children and How they learn?

Let us for a moment go back to when you were a child and simultaneously keep your present classroom and the children you teach in mind. You will agree that each child is different from the other. Each child has her/his likes, dislikes, interests, skills and way of behaving. Thus, every child is unique in her/his own way. Since every child is an unique individual, she/he learns and responds to situations in her/his own special way.

While assessing children it is important to appreciate differences amongst them and respect the fact that they will understand and respond in different ways while learning. You would also have noticed that when a child enters Class 1, she/he brings to school a lot of different experiences and some kind of a knowledge base and vocabulary. Children are not '*empty vessels*' or '*blank slates*', to be filled up with information and knowledge that only the school can give, as is generally believed. It is important to build on experiences that a child brings to school. New learning is thus to be developed based on what the child already knows and understands.

At the same time, it is also necessary to understand how children learn at the primary level as this in turn will also determine how each child can be assessed during the teaching-learning process. Some important aspects are that–

- every child can learn if allowed to do so at her/his own pace and follow her/his own way of learning.
- children learn more through play/activities and learn better from each other and if they actually *'do'* things.
- learning is a continuous process. Thus, children's learning does not take place only in the school. Therefore, classroom learning should be linked to what happens outside the classroom and at home.
- children *'construct'* their own knowledge and do not learn only when and what the teacher teaches. This means that every child makes sense of what information he/she is exposed to based on his/her previous experiences and learning. Only then does the child arrive at his/her own understanding and conclusions. Each child has a unique approach to acquiring knowledge. This process goes on continuously.
- children at the primary stage learn better and more easily through experiences, play, exploration, trying out various things and actually *'doing'* different activities.
- children learn in a spiral and not a linear way. Thus revisiting concepts again and again helps them to understand better. The act of learning involves a process of establishing connections among facts observed/experienced or realised by children. The new learning, therefore, is to be based not only on the preceding facts and information but could be related even to things acquired long back in school, home or elsewhere. Hence, learning does not proceed in a linear manner.
- children learn through the mistakes and errors they make.
- learning takes place in a holistic manner, thus, an integrated approach to learning is better.

It has been seen that children in the primary classes learn better while interacting with others, more so if they are encouraged. Their learning is promoted much more through play, imitation, practice, proceeding from concrete to abstract, simple to complex and if the entire learning process is joyful and stimulating.

Understanding the School and the Classroom

Teaching-learning in the classroom depends, to a large extent, on the school environment and ethos. A safe, secure and happy school environment can help children learn better and achieve more. For this, it is necessary that the school makes available necessary facilities such as learning materials, aids, equipment and space for doing activities, working together and playing. Children's learning in primary classes is promoted much more through play-way methods and a child centred approach. What does the term *'child-centred'* imply? Let us look into two classrooms – one that is teacher-centred and the other child-centred so that the term used becomes clearer.

A Picture of Two Classrooms

Teacher-centred Classroom

- Teacher gives instructions and expects children to obey and be disciplined.
- Children listen while the teacher teaches.
- Teacher reads the textbook or writes questions and answers on the blackboard and students copy these. At times, one child reads the textbook aloud while others listen.
- Children memorise facts given in the textbook or as told by the teacher.
- Teacher controls what happens in the class. children's participation is minimal.
- Children generally learn individually.
- Timetable is fixed.
- Seating arrangement is fixed.
- Material is only for display.
- Children look bored and disinterested.
- Assessment is undertaken as a seperate activity.
- Children are assessed through tests/exams.
- Report card conveys the child's achievement.
- Children's performance is reported in terms of marks for subject areas.

Child-centred Classroom

- Teacher provides learning opportunities and guides a range of meaningful learning.
- Teacher provides learning situations that give children an opportunity to observe, explore, question, experience and develop their own understanding of various concepts.
- Children participate actively in different activities/tasks.
- Children construct knowledge on their own, based on their experiences inside and outside the school.
- Children work both individually and also in groups, discussing, sharing, co-operating and respecting others' viewpoints.
- Timetable is flexible, depending to a large extent on what children want to do.
- Seating arrangement changes according to the activity being organised.
- A variety of materials, aids and equipments are available and used by children.
- Children are engrossed in what they are doing.
- Assessment is part of the teaching learning process.
- Children are assessed while doing activities/tasks.
- A report conveys the learning and progress of the child.
- Children's progress is reported in qualitative terms and on all aspects of development.

What emerges from the picture on the previous page is the kind of classroom we would like to see, namely, a child-centered classroom. In this kind of classroom assessment would be–

- child-centred and that which takes into consideration differences between learners.
- catering to each child's needs, pace and style of learning.
- flexible, need based and appropriate to the child's age and level and
- a part of the teaching-learning process, continuous and comprehensive.

Why should Children be Assessed?

Since all of us are concerned about children's learning and providing quality education to all children, the reasons as to why assessment is undertaken in primary classes will be varied. You may already be aware of some and may also be carrying out some during teaching-learning. Some of the important reasons to assess are to

- find out what learning, changes and progress takes place in the child over a period of time in different subjects and other dimensions of her/his personality.
- identify individual and special needs and requirements.
- plan teaching-learning situations in a more suitable way.
- help the child understand and slowly realise what she/he can or cannot do, what interests them and what they like or do not like to do.
- find out to what extent curricular expectations and syllabi objectives have been achieved.
- improve teaching-learning processes in the classroom.
- provide evidence of children's progress so as to communicate the same to parents amongst others.
- do away with the fear of assessment (prevailing examination) among children and ultimately encourage each one to assess herself/himself.
- support and improve every child's learning and development.
- encourage a feeling of confidence and accomplishment amongst children.

What should be Assessed?

The obvious question that most teachers ask is – what aspect of children's education or learning should be assessed? We need to ask ourselves what is it that we are looking for when we assess children? Since education is concerned with the total (all-round) development of the child (physical, socio-emotional along with cognitive) and schools support and encourage this, all aspects need to be assessed rather than only academic achievement, which is presently the focus of most assessment practices being followed in most schools. It is, therefore, important that assessment be undertaken for a whole range of activities that the child participates in both inside and outside the school-classroom. This would enrich and make the process more comprehensive in nature. The assessment process needs to be a way of providing information and feedback on the extent to which the school and teachers have been successful in imparting quality education to every child.

In order to arrive at a complete picture of the child's learning and total personality development assessment should focus on

- children's learning and performance in different subject areas, behaviour and progress over a period of time.
- different dimensions of children's personality development over a period of time.
- children's skills, interests, attitudes and motivation amongst other aspects.
- children's responses to different situations and/or opportunities both in and out of school.

When should Assessment be Done?

Closely related to what needs to be assessed is the critical question expressed by most of us as to when or how often should a child's learning and progress be assessed? Assessment of the outcomes of learning goes along with the teaching-learning process in a continuous manner. In order to undertake a holistic assessment, all aspects of learning need to be given due recognition. The manner and modalities however may vary. While teachers are regularly observing the progress of children some periodicity would be necessary. It implies maintaining a profile for each child. This is required in order to reflect upon, derive feedback, plan and implement measures to enrich and enhance children's learning. This will call for a judicious cycle to be followed. One cannot undermine the fact that while informal observations continue, fortnightly looking back and quarterly reviews are generally recommended to promote and enhance learning among children. Thus assessment may be on

- **Daily basis** – interacting with children and continuously assessing them both in situations inside and outside the classroom.
- **Periodic** – once in every 3 to 4 months, teachers may check and reflect on the information collected. This however should not be in the form of a test/exam but for purposes of reflection.

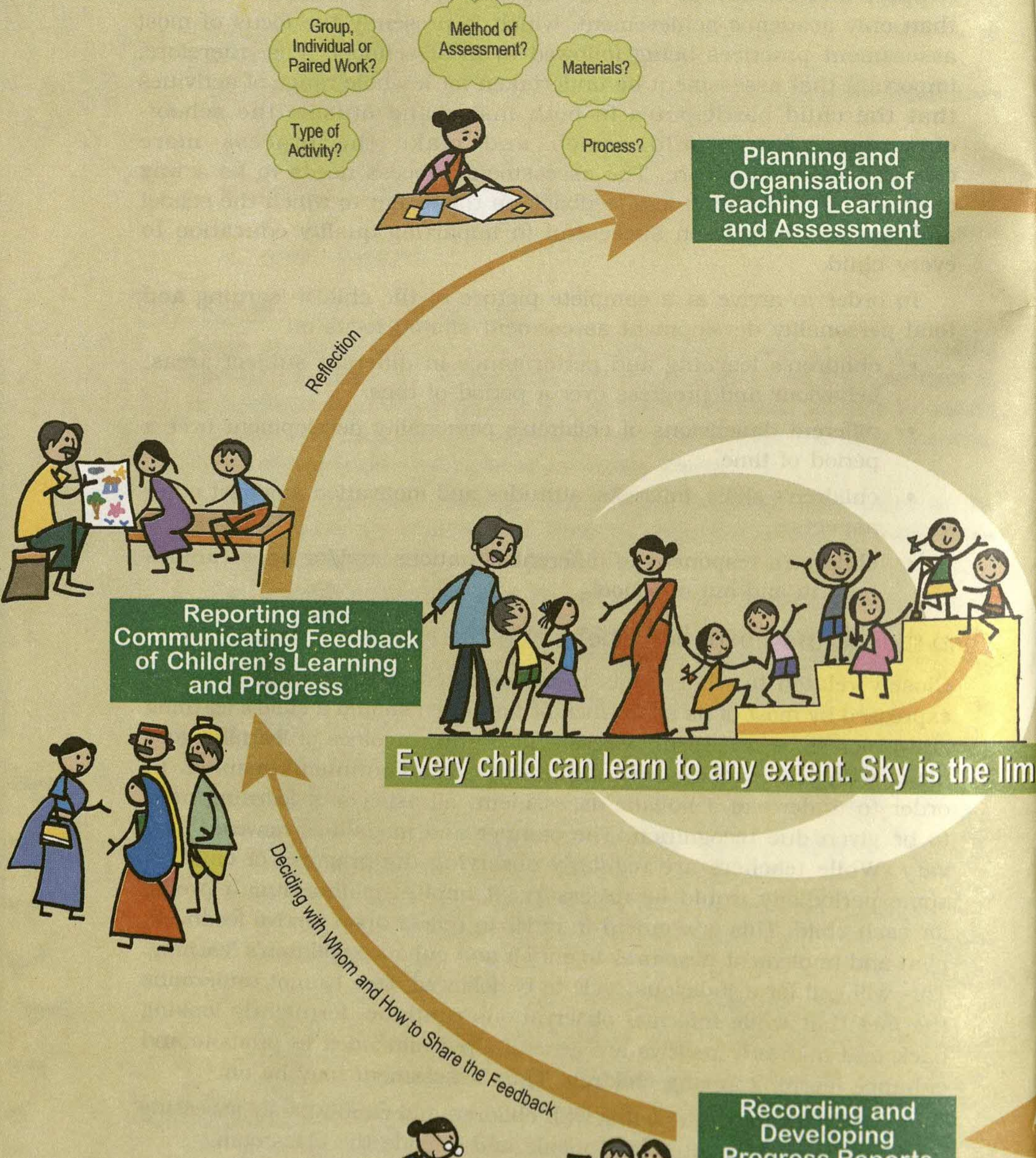
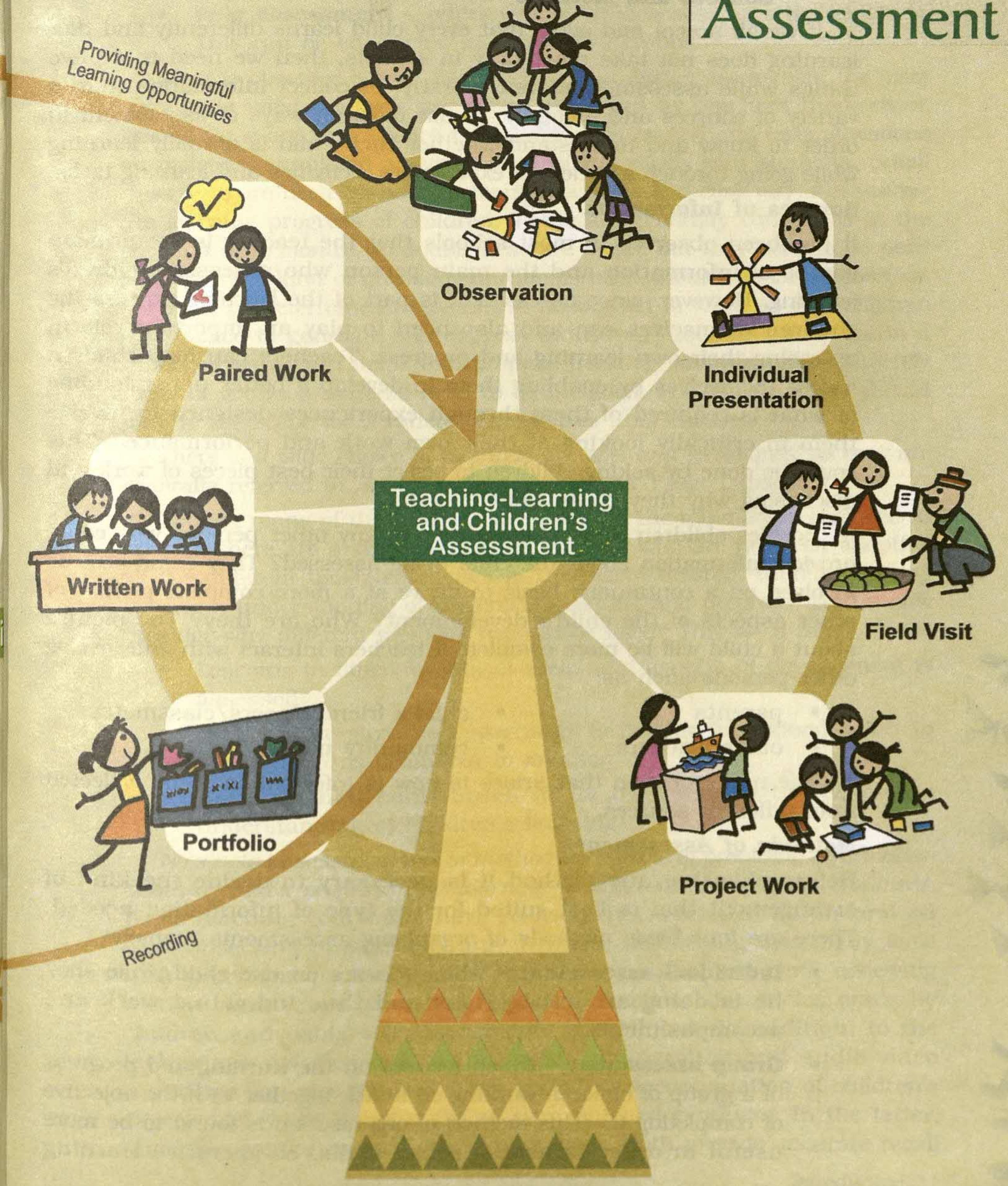

Group, Individual or Paired Work?
Method of Assessment?
Materials?
Type of Activity?
Process?
Planning and Organisation of Teaching Learning and Assessment
Reflection
Reporting and Communicating Feedback of Children's Learning and Progress
Every child can learn to any extent. Sky is the lim
Deciding with Whom and How to Share the Feedback
Recording and Developing Progress Reports

Figure 1

Cycle of Learning and Assessment



How should Assessment be Done?

The various steps and process of assessment which is cyclic and continuous is detailed below and also presented in Figure 1 *(pages 8-9)*

Step I Collecting Information and Evidence through Different Sources and Methods

If we all accept and agree that every child learns differently and that learning does not take place only in schools, then we need to do two things while assessing children – firstly, to collect information from a variety of sources and secondly, to use different ways of assessment in order to know and understand whether each child is actually learning while going through a variety of experiences, activities and learning tasks.

Sources of Information

It has been observed in most schools that the teacher is the primary source of information and the main person who assesses children's learning. However, since assessment is part of the learning process the children themselves can and also need to play an important role in assessing their own learning and progress. Teachers can help children assess themselves by enabling them to develop a better understanding of what is required of them through experiences designed to involve them in critically looking at their own work and performance. This could be done by asking children to select their best pieces of work and discussing why they chose what they did.

Besides children themselves, are there any other persons who could provide information about the child being assessed? They could also be involved on a continuous basis to arrive at a more complete picture of other aspects of the child's development? Who are they? The picture about a child will be more complete if teachers interact with and involve other persons such as:

- parents
- other teachers
- child's friends/peers/classmates
- community members

The next question that arises is how is information to be collected from different sources.

Methods of Assessment

Before choosing any method it is necessary to decide the kind of arrangement that is best suited for the type of information needed. There are four basic methods of organising assessments, namely:

- **Individual assessment** – which focuses on one child while she/he is doing an activity/task and thus individual work and accomplishnents.
- **Group assessment** – which focuses on the learning and progress of a group of children working on a task together with the objective of completing it. This method of organisation is found to be more useful in order to assess social skills, co-operative learning

processes and other value related dimensions of a child's behaviour.

- **Self assessment** – refers to the child's own assessment of her/his learning and progress in knowledge, skills, processes, interests, attitudes etc.
- **Peer assessment** – refers to one child assessing other children. This can be conducted in pairs or in groups.

What is observed across all schools is that during assessment the most commonly used methods are those developed by teachers themselves. Among these are paper-pencil tests/tasks, written and oral tests, questions on pictures, simulated activities and conversations with students. Small class tests are used by most teachers as a quick and easy way of assessing the learning progress of children. These are generally conducted at the end of a unit/month. No doubt these are useful but they need to be used carefully. The kind of questions and items used should, as far as possible, not have predetermined answers but be worded in ways so that children have space to generate and express individual thoughts and ideas in a variety of ways. Test items that promote thinking, analysis and not merely recall of textbook material need to be included. Put simply, items should provide the scope for a variety of responses from children.

There is a wide choice of methods or '*tools and techniques*' as they are generally referred to, which is being shared in Table 1 (*pages 12-13*).

Though many of the methods presented in the table are being used in schools, there are a large number of teachers who are not seen adopting all the different tools/techniques to the extent desired, thereby limiting their own understanding of the child's learning and progress. Let us now consider why different methods need to be used? This is so that

- Learning in different subject areas and aspects of development is assessed,
- Children are given an opportunity to be able to respond better to one method as compared to another,
- Each method contributes in its own way to the teacher's understanding of children's learning.

No single assessment tool or method is capable of providing information about a child's progress and learning in different areas of development. While teaching you would have realised that a lot can be understood from observing pupils, listening to them, discussing informally with their peers and parents as well as talking to other teachers, reviewing written work (class work and homework) and other articles made by children and using self-assessments by children. In addition, to the techniques detailed out in the table, photographs and audio-video recordings can also be used. They provide a documentation of children's experiences while doing tasks or even finished products. In the latter, processes can be covered over a time span. Both provide accurate recall

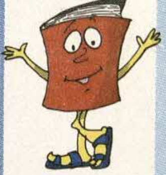
Table 1: Techniques for Assessing Children's Learning: The Choice Available

Types of Assessment tools and techniques	What are the Strengths and Advantages?
1. Observation Information can be gathered about children in 'natural' settings.Some are based on teachers' observations about childs in the course of teaching. Others are based on planned and purposeful observation of children on activities/tasks.	• Various aspects of personality development can be assessed through observations. • Can be used to assess individuals as well as groups. • Assessments can be made during varying time periods. • Evidence of child's performance/knowledge is based on an 'on-the-spot' record. • Over time, detailed observations of behaviour as well as interests, challenges, – patterns/trends emerge which allow teachers to create a comprehensive picture/view of the child.
2. Assignments Written assignments are designed to allow the child to plan, compose and report about a unit/area/theme of learning. These can be completed as class work and/or homework. Can be open ended or structured. Some could be based on contexts outside textbooks as well. Presentations can also be used on an individual or group basis.	• Provides children an opportunity to search for information, construct their own ideas, and articulate the same ideas through spoken, written and/or visual expressions. • Helps assess a wide range of objectives and content of learning. • Provides children an opportunity to relate and synthesise within and outside school learning.
3. Projects These are undertaken over a period of time and generally involve collection and analysis of data. Projects are useful in theme-based learning.	• Provides opportunities to explore, work with one's hands, observe, collect data, analyse, organise and interpret data and draw generalisations. • Provides an opportunity to work in groups and in real life situations. • Helps develop a positive attitude towards group work, sharing and learning from each other.
4. Portfolio Collection of a child's work over a period of time. It could be day-to day work or selection of the child's best piece of work.	• Provides a cumulative record. In the process, a picture of how a skill or knowledge area develops emerges. • Enables the child to demonstrate to others, his/her learning and progress. • The child becomes an active participant in learning and assessment.
5. Checklists A systematic way of recording specific behaviour/action helps focus attention on particular aspects. Observation checklist records the presence or absence of a particular knowledge item, skill or process. Checklists are a list of criteria that the teacher determines are important to observe in a child at a particular time.	• Quick and easy to implement. • Provides specific information about specific objectives. • Can point towards a trend of how and when skills have been acquired by the child as well as a group of children.
6. Rating Scales Rating scales have the same usage as observation checklists. They record the degree to which the presence or absence of a particular knowledge items skill of process are found or quality performance. Used to record and judge the quality of a child's work against specified criteria. Holistic rating scales require a single, overall assessment of a piece of work.	• Various aspects of development that can be described along a continuum can be assessed. • Can be used to assess individuals as well as groups. • Assessments can be made during varying time periods and in different environmental settings. • Evidence of the child's performance/knowledge is based on 'on-the-spot' record. • Over time, detailed observations of behaviour as well as interests, challenges,– patterns /trends emerge which allow teachers to have a comprehensive picture/view of the child.
7. Anecdotal Records Refers to written descriptions of a child's progress that a teacher keeps on a day to day basis. Provides observational narrative records of significant incidents in a child's life.	• Provides a wealth of information across different developmental areas. • Facilitates taking of notes on the child's social and emotional development, choices, interests and relationships etc. • Identifies strengths and weaknesses in the child. • Helps assesses children's progress over time.

Note

This table does not provide an exhaustive list of techniques. No single technique can provide all the required information or evidence about a child's learning or progress. They can be used in different combinations and at different times depending on what it is that is being assessed.

What should be taken care of?	How can more value be added?
• Avoid arriving at inferences/interpretations or jumping to conclusions. Important to take down more than what is actually seen. • Dependent on the skill of the observer which determines 'what' is observed. • Requires sensitivity and unobtrusiveness in the way the observation is done. • Observations are to be made over a period of time, across different activities and settings.	• Recording details that not only describe the actions but reveal how a child feels about what she/he is doing, details on how she/he does something as well as when she/he does it. The quality as well as the quantity of her/his interrelationships with people and materials, and what he/she says etc. • Noting comments about the child's behaviour in parentheses based on which processes can be inferred at a later point of time.
• Not too much homework or class work should be given which is currently the normal practice. • Assignments should be such that they can be managed by children on their own. • Should not become the only method of assessment.	• Going beyond collection of assignments by following it up with analysis, discussion and reflection. • Creativity on part of the children is to be promoted. • Encouraging children to go beyond textbooks. • Group work is to be encouraged. • Can become part of the portfolio.
• The nature and difficulty level of the projects should be such that children can do it by themselves. • Materials to be used for the project should be available in the school, neighbourhood or home setting. These should not put a financial burden on the parents. • Each school could go in for a Resource Centre, which would have locally available materials.	• Project topics should be decided/chosen, planned and conducted by children largely with the teacher acting as a guide. • Encouragement should be given to group projects. These will enable children to work together, share experiences and learn from each other. • Projects keep giving children an opportunity to explore, investigate and work in groups.
• Selecting work to be put into the portfolio should have a specific reason. • Not all papers/items of work are to be included. This will become unmanageable.	• Children's participation in selecting portfolio contents is to be encouraged, as well as criteria for selecting the contents. • Continuous updating of the portfolio as the child grows. • Careful structuring of portfolio material accompanied by a reflective account. • Clear labelling and numbering of content for easy reference.
• Limited information, only indicating presence of a skill. • Does not indicate the child's response to different situations or provide specific examples of responses. • Does not provide information about context. • Can at times become unwieldy because of the number of specific items. • If developed by others, may not be suitable for the objectives that you as teachers have in mind or, for the group, you wish to use it with	• Add a 'comments' column to add value to the information in the checklist marking. • Use this tool in conjunction with other methods of assessment.
• Avoid inferences/interpretations or giving judgments. Concentrate on taking down what is seen. • The skill of the observer may determine what is observed. • Be sensitive and unobtrusive in the way the observation is done. This does not necessarily mean being at a distance (physically). • Make the observations over a period of time, and across different activities and settings.	• Record details that not only describe the actions but reveal how a child 'feels' about what she/he is doing. • Suggest corrective measures. • Comments can be noted in 'parentheses' based on which processes can be inferred at a later point of time.
• A single anecdote does not give conclusive information. • Only 'problematic' situations may be noticed. It would be better to describe incidents rather than making statements of judgment. • Selecting amongst the many interesting classroom events and not including all. • Avoiding general comments.	• Preparing and collecting anecdotes over a period of time about sustained interests and critical things that are happening in a child's life. Helps to understand the child's behaviour/responses to different classroom situations. • Collecting a cross section of anecdotes [from different children] indicates groups' thinking and feeling. • Recording as soon as possible after the event so that rich, accurate and significant details can be included for later interpretations.



of events and an insight into the child's way of thinking and interaction. It also facilitates sharing of information with both the child and parents. However, these are expensive, require technical expertise, and analysis which is also time consuming.Thus they need to be used very carefully.

Step 2 Recording of Information

In schools across the country, the most common form of recording is through the use of report cards. Most report cards carry information either in the form of marks or grades obtained by children in tests/ exams. The usefulness and implication of marks and grades has already been discussed in the beginning of the chapter. The crucial question that arises is what can be done to improve the process of recording. If the central purpose of continuous assessment is to find out the child's level of learning in a particular subject then it becomes necessary to make and record observations of the child while she/he is on the task or involved in an activity.

Making Recording more Effective

- Observing children and immediately recording the observations, changes seen, in a diary or a register or a note book etc.
- Assessing the child's work during an activity or when it is completed.
- Making a special effort to write qualitative and descriptive statements of a child's work or any interesting incident.
- Preparing a child's profile.
- Keeping samples of a child's work in a portfolio.
- While making notes talking to the child about what is being done and how it is being done.
- Making a conscious effort to note down important changes, problems, strengths and learning evidences.
- Clarifying any doubts with the child while recording.

Take Care: Biases or Errors can be made that affect recording

It has been observed that many a times errors could be made while observing children's learning and progress. These are the result of certain biases such as:

- Previous impression of the child's ability, potential and performance.
- Viewing the male child more favourably than the girl child or vice-versa in some situations.
- Viewing one aspect of the child's work based on her/his earlier performance in another subject area.
- The background the child comes from caste, class, community, location, etc.
- Previous test results in one area of learning in one of the subjects.
- Tendency to give a similar rating to a criteria close to another criteria in the same subject.

NOTE

It is important to give full attention to what is being observed in the child.

Classroom interaction provides a wide range of opportunities to make observations of a child's behaviour and learning. As you are aware some of the observations are made on a daily basis in an informal manner while teaching-learning is going on. Day-to-day observations are easily forgotten if not recorded. Still others are planned observations of children on activities given to them. This type of observation is planned with a purpose and is thus more formal in nature.

To provide a more complete picture of the child's learning and progress, the scope needs to be widened. Recording needs to include records of observations and comments on children's performance on assignments, ratings of what children do and how they behave and anecdotes or incidents of children's behaviour towards others.

It is important that the information collected should be understood and a variety of responses both encouraged and appreciated by the teacher. A number of examples and illustrations are given in the chapters that follow.

Step 3 Making Sense of the Information Collected

Once information has been recorded, the third important aspect or the next step is using the available evidence to arrive at an understanding of what has been collected and recorded. What this results in is drawing conclusions about how a child is learning and progressing. This is necessary in order to understand *'where the child is'* and *'what needs to be done to help the child'*. This requires daily analysis and review of records as well as periodic reflection of collected information. All this will help teachers to reflect on their teaching practices, classroom management, and use of materials amongst other pedagogic aspects and improve on the same for the benefit of the learner. Proper interpretation would require identification of indicators to facilitate the process.

Indicators – A Frame of Reference

In order to make the analysis easier a set of indicators have been developed in the subsequent chapters in the *Source Book* for Mathematics. These have been worked out for each class/level and draw substantively from the objectives of learning for the subject area as framed in the syllabus of NCERT at the Primary Level based on the NCF-2005. However, **these are only provided as a broad frame of reference. You may like to adapt them as per your state syllabi in general and local requirements in particular.**

Why are indicators/ markers viewed as important?

Indicators should help in a number of ways by

- focusing and understanding children's learning better on a continuum of learning.
- providing a reference point for parents, children and others to understand the progress of every child in a simple way.
- providing a framework for monitoring, learning and reporting progress about the child.

It is very important that information which is collected should not stop with collection of information/ evidences. You need to carry this further and indicate with brief qualitative remarks or other ways what it is about a given response that makes it more or less appropriate, acceptable, significant or interesting. It often happens that a child's response is marked with a 'O' or 'X'. Alternatively 'A' or 'B' etc, is given, at times with no further interaction with the child. It is important and necessary to go beyond marking or grading responses like right or wrong, if assessment data is to enrich teaching-learning practices and the child's learning. This will help in understanding *why* the child has done *whatever* she/he has attempted to do by way of explanation. The following chapters in the *Source Book*, discusses all this in detail, with illustrations. After interpretation you need to plan and focus on how and with whom you should communicate and share the feedback on assessment.

How can Assessment Information be Used?

Reporting and Communicating Feedback on Assessment

When assessment is undertaken it will result in some amount of information with the teachers about the child. Once this information has been recorded and analysed what is to be done with it. You will agree that generally across all schools information on assessment of a child's learning and progress is conveyed to both the child and parents through a Report Card. This is supposed to present a picture of a child's performance in different subjects in the form of marks/grades generally, based on tests/exams conducted periodically in a school year.

There is no doubt that the assessments made by teachers and the records they keep are done so as to help them understand how much children have learnt, improve their teaching-learning processes and provide more meaningful learning opportunities/experiences to further enhance the learning of each child. In order to achieve the above, reporting needs to become more communicative, constructive and user friendly. This would be possible if teachers reflect on what information they have with them through their daily experiences and vis-à-vis the indicators in a specified area of learning.

Reporting

Reflection by the Teacher

Daily as well as periodic assessment will only help if you

- assess the portfolio and other records on a periodic basis i.e. quarterly (once in every 3 months).
- review interesting incidents and assess the other aspects of the child's personality.
- compare it with earlier records.
- ensure that the same problems have not come up.
- look at how the problems and difficulties have been taken care of.

- assess whether the child has improved and if any weaknesses still remain, if so what action needs to be taken in the teaching-learning situation.

Teacher's reflection will help in preparing the progress map viz. a cumulative report that provides a clear picture of a child's progress over a given period of time. Only then can they meaningfully guide children's learning in the future and help them to progress from a lower level of understanding and skill acquisition to higher and more complex levels of learning. It will also help identify what it is that the child finds difficult and address the gaps. It is through this feedback that changes can be made in the teaching-learning process.

Once the feedback is generated, the critical question that needs to be addressed is what should a report that is made by the school-teacher contain. **It should provide a profile of the child's progress over a specified period of time**. How can a child's progress be described? Let us consider how this can be done and what information needs to be included.

Report: Mapping the Progress of the Child by

- assigning grades A,B,C in the subject areas. These grades will indicate the range within which the child's learning and performance lies in the three performance bands or levels.
- using a collection of illustrations of a child's work to help understand at what level of learning she/he is.
- focusing on different aspects of the child's personality.
- providing qualitative statements about what and how the child is learning along with grades.
- providing examples of the kind of work that has been done by the child.
- highlighting the strengths and also those aspects requiring further improvement.

On preparing a report the teacher needs to communicate and share the feedback with the child and parents. This aspect is important and needs to be done carefully and in a constructive and positive manner.

Communicating Feedback on Assessment

Sharing with the Child

On a daily basis most teachers do provide informal feedback to the child while she/he is involved in a task/activity. Children also correct and improve themselves while observing the teacher, other children or while working in pairs or groups. The problem arises when children are given a report in such a manner that it reflects what they cannot do, their failures or inadequacies. This demotivates children. The teacher needs to

- discuss with each child her/his work, what has been done well, not so well and what needs improvement.

- jointly (teacher and child) identify what kind of help the child needs.
- encourage the child to visit/see her/his portfolio and compare the present work with what was done before.
- provide positive constructive comments while the child is working or on the work already done.

More importantly, what needs to be encouraged through feedback is for the child to **compete with herself/himself rather than what others are doing**. It should be with reference to – 'Where was I yesterday or a week ago and where am I today'? Comparisons between children do not really help. By and large it leads to feelings of '*I am no good*' or if a child has done very well and got the highest marks, he/she is put under pressure to keep up the performance at school and by parents at home.

Sharing with Parents

Parents are likely to be the most interested in knowing how their child is '*doing*' in school, what she/he has learnt, how is their child performing and what is the progress of their child over a given period of time. More often than not, teachers feel they have communicated effectively through comments made to parents such as – '*can do better*', '*good*', '*poor*', '*needs to put in more effort*'. For a parent what do these statements mean? Do such statements provide any clear information of what their child can do or has learnt etc. As a concerned teacher in order to enrich the feedback being communicated, it is suggested that you focus in simple and easily understood language on–

- What the child can do, is trying to do and finds difficult.
- What does a child like or not like to do.
- Qualitative statements and quantitative feedback with samples of the child's work.
- How the child has learnt (processes) and where did she/ he face difficulty.
- Whether he/she could complete the activity and his/her performance while doing so.
- Sharing the child's work with parents, to help indicate areas of success and improvement.
- Talking on aspects such as cooperation, responsibility, sensitivity towards others, interests, etc. with both the child and parents.
- Discuss with parents (a) how they can help (b) what they have observed at home about the child.

You may like to think of presenting the child's progress through a graph or drawing which may make it easier to show a child's progress to the parents and others.

All the information gathered about children's learning and progress and the feedback gained, should ultimately enrich the ongoing teaching-learning process further and help children learn better. This calls for further reflection by the teachers. In order that the assessment cycle continues in a useful manner, teachers may also like to consider reflecting on their own teaching-learning practices, use of TLM, planning and providing tasks/activities amongst other aspects based on the assessment of children in different learning areas.

Reflection by the Teacher for Enriching Children's Learning

Some key questions that may help in reflecting and also discussing children's progress with others are:

- Are my children involved in activities and learning optimally? If not, at what level are most of the children?
- Am I able to understand the different needs of children? If yes what am I doing to cater to those needs?
- Are there any children who are finding it difficult to reach even the first level? What should I do to motivate and encourage them individually?
- How can I improve my teaching-learning practices so as to bring children from one level to the next?
- How can I motivate children towards self-assessment?
- What are the difficulties facing me? (Dividing children into groups, finding appropriate activities according to the age and level of the child, insufficient materials for all children etc.)
- What further help do I need? Who can offer this? (educational functionaries, parents, community, other teachers).
- What effort should I make to arrange better teaching-learning practices?

You may like to think and ponder over these and other such questions that will help you to plan better teaching-learning strategies for the future.

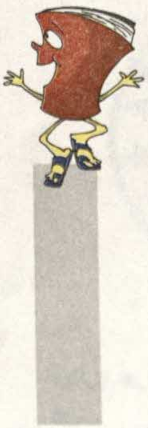
It is Possible– You can make it Happen

Assessment can become a useful and interesting process. To realise this you need to be careful about:

Being clear about why you are assessing the child.

Not labelling children as, slow, poor, intelligent, disturbing and making comparisons between children.

Using a variety of ways to collect information about the child's learning and progress in subjects and crossing curricular boundaries.



Collecting information continuously and recording the same.

Giving importance to each child's way of responding and learning and the time it takes to do so.

Reporting on an ongoing, continuous basis and being sensitive to every child's responses.

Not making negative statements and using technical language during assessment or while providing feedback to the child, parents or others.

Providing feedback in clear and simple language, which will lead to positive action and help the child to do better.

The chapters that follow will further help in assessing children in Mathematics at the primary level, within the approach so far discussed.

2

Mathematics

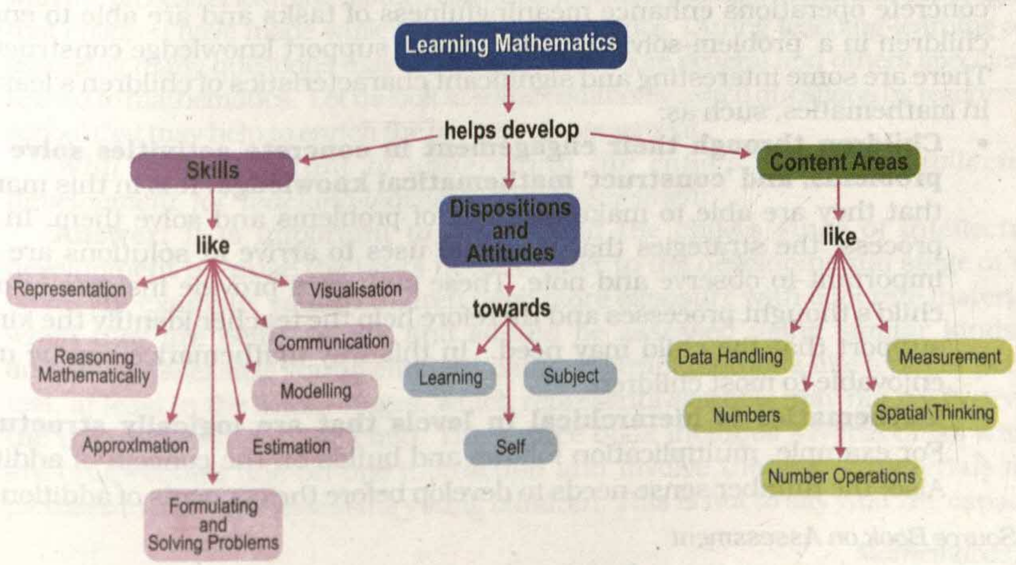
As you turn my pages, you will find activities and ideas for a more child-friendly assessment.

Teachers will agree that formal assessment is a way of finding out what the students have learnt. Equally important is that this is also a way to understand the efficacy of their own teaching methods. In this chapter we provide you with suggestions so that as a teacher it would be possible for you to find out what students have learnt and also

- do this through child friendly, interactive methods in both formal and informal settings.
- develop a more holistic understanding of the child's learning by using a variety of activities and tasks in creative interaction with children.
- record, understand and interpret the observations made during assessment.
- develop ideas on what needs to be done next that can be a part of your future teaching plan.
- provide feedback to children and in the process also enable them to become responsible for their own learning.
- assess the effectiveness of your classroom practices so that you can make necessary modifications.
- give you some basic ideas so that you are able to make your own activities and items and observations to improve the assessment process.

2.1 Learning Mathematics

In the primary school years, the aim of school mathematics is to develop 'useful' capabilities and also to develop the ability to think and reason mathematically.



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'Useful' capabilities include conceptual understanding and ability to understand and solve problems in the areas of numbers, number operations, measurement, spatial thinking and data handling. Resources for mathematical thinking and reasoning include the ability to reason mathematically, formulate and solve problems, make estimations, find approximate solutions, pursue assumptions to logical conclusions and abstract reasoning. While learning mathematics, children develop and express self confidence, creativity, ability to solve mathematical problems and ability to communicate and use mathematical concepts and symbols.

"Having children develop a positive attitude towards and a liking for mathematics at the primary stage is as important, if not more so, than the cognitive skills and concepts that they acquire. Mathematical games, puzzles and stories help in developing a positive attitude and in making connections between mathematics and everyday thinking. It is important to note that mathematics is not just arithmetic. Besides numbers and number operations, due place must be given to shapes, spatial understanding, patterns, measurement and data handling. The curriculum must explicitly incorporate the progression that the child makes from the concrete to the abstract while acquiring concepts. Apart from computational skills, stress must be laid on identifying, expressing and explaining patterns, on estimation and approximation in solving problems, on making connections and on the development of skills of language in communication and reasoning" (NCF 2005).

2.2 Children Construct Mathematical Knowledge

All children bring to school a wide range of experiences since mathematics is a part of everyday activities. Those especially from the rural areas, come to school with knowledge that is rich in oral mathematical traditions including techniques for computation, mathematics in riddles and recreation and even problem solving. Children develop mathematical thinking through everyday interactions in the world. These are resources that children already have access to, and which can be a part of the classroom processes.

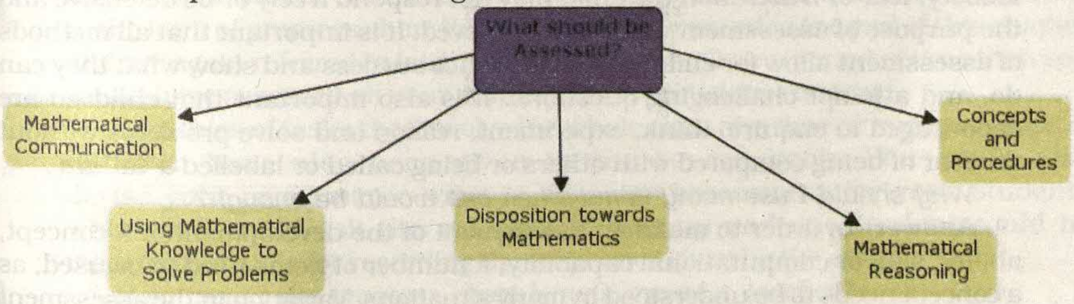
School is a place where the child is introduced to the more formal aspects of mathematics. The aim of the school is to give children opportunities to develop their mathematical understanding through engagement with a variety of 'concrete' and 'contextualised' learning activities and reflection of the same. Contexts and concrete operations enhance meaningfulness of tasks and are able to engage children in a 'problem-solving mode'. These support knowledge construction. There are some interesting and significant characteristics of children's learning in mathematics, such as:

- **Children through their engagement in concrete activities solve real problems, and 'construct' mathematical knowledge.** It is in this manner that they are able to make meaning of problems and solve them. In this process, the strategies that the child uses to arrive at solutions are very important to observe and note. These strategies provide insights into the child's thought processes and therefore help the teacher identify the kind of support that the child may need. In this way mathematics can be made enjoyable to most children.
- **Mathematics is hierarchical in levels that are logically structured.** For example, multiplication follows and builds on the concept of addition. Also, the number sense needs to develop before the concepts of addition and

subtraction. Once the child has understood the concepts of addition and subtraction, this in turn helps strengthen the concept of numbers.

- **Children learn in stages.** It is at the primary stage that the base for further learning is laid. In the course of learning children develop concepts, procedures and skills, all of which can be considered dimensions of mathematical knowledge, and support each other's development. For example, in learning about measurement, children develop their concepts of measure from their concepts of comparison (bigger-smaller, longer-shorter etc.). In addition they also develop skills of measuring with instruments.
- **The experiences children have while learning mathematics directly influence their motivation and ability to learn.** The ability to learn mathematics is also influenced by the child's attitude towards himself or herself as a child, and towards the subject matter. Keenness and eagerness to solve problems, perseverance in problem solving and difficult tasks, and enjoyment of the beauty of mathematics – all these are also aspects of learning mathematics that are influenced by the teacher in the classroom.
- **In view of how children, in primary classes, learn mathematics, the focus of assessment would be on**
  - understanding of how children learn mathematics;
  - understanding of the mathematical concepts; and
  - understanding the child's understanding of mathematics.

Let us now reflect on what needs to be assessed broadly in mathematics. These areas are presented in the diagram below.


What should be Assessed?
Mathematical Communication
Concepts and Procedures
Using Mathematical Knowledge to Solve Problems
Disposition towards Mathematics
Mathematical Reasoning

The effort in this Source Book is to present ways of assessing the child that are consistent with the process of learning mathematics and to enable teachers to monitor the progress that children have made while learning mathematics. During teaching-learning and assessing many questions may arise – some are more general and others specifically related to mathematics. Let us look at some commonly asked questions by teachers in school that may help to enrich the process of assessment.

Why is it important to understand the child's stage of intellectual development for teaching-learning and assessment?

Answer: It is important to understand the child's stage of intellectual development because the tasks need to cater to the developmental stage of the child. For example: young children need more exposure with concrete materials when they are being taught and should be assessed using similar kinds of activities. Assessing young children using abstract methods like a paper pencil test, at least in the initial stages, which represents symbols may not really reveal the understanding of the child. The source book includes several tasks which are oral; require one-on-one interaction and involve the use of materials and pictures to facilitate assessing young children. This is not to say that the capacity

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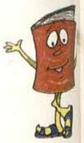
for more abstract reasoning and symbolic manipulation is excluded but this should certainly not be the exclusive method of assessment.

Some children of my class can solve everyday problems orally but are unable to apply the same understanding when it comes to answering in a test. Why is it so?

Answer: Children find it meaningful to solve problems that are real-life and are about things that concern them. Therefore it is important that we as teachers provide ample opportunities to students to show their mathematical understanding independent of school-learning which is based on symbolic representation most of the times. The problems therefore need to be embedded in the context of the children.



Why is it important to assess the child's previous knowledge before moving on to the next concept?

Answer: This has connection with teaching. Assessment is done for what is learned, thus the decision about the prerequisite has already been made. Mathematical concepts are often hierarchically built. Analysis of the prerequisites can feed meaningfully into teaching process to ensure that children are able to learn what they are ready for rather than having their confidence destroyed by trying to teach them what they are not yet ready for. Analysis of assessment, especially error analysis, can contribute to identify such learning gaps.

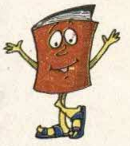
Why should a friendly disposition be maintained while assessing children?

Answer: The purpose of assessment is to understand the learning level of the child and provide support wherever she/he needs it. If assessment leads to anxiety, fear or frustration the child may not respond freely or be defensive and the purpose of assessment will not be achieved. It is important that all methods of assessment allow for children to experience success and show what they can do, and attempt challenging questions. It is also important that children are encouraged to enquire, think, experiment, reason and solve problems without the fear of being compared with others or being called or labelled a 'failure'.

Why should I use many items when one would be enough?

Answer: In order to make an assessment of the development of a concept, ability, skill or computational capability, a number of items need to be used, as a concept needs to be understood in many situations. Variation in the assessment items is also essential as they provide children with wider range of opportunities to demonstrate their conceptual understanding. For example, if one wishes to assess children's conceptual development of number and addition, just asking a child '53+28' is not sufficient. More items on addition '87+56, 77+34, 13+39, 85+15' may provide us with understanding that they are able to add these numbers where the sum of tens digits provides slightly different conditions where the carry over is to be added. When a number of items are used they may be administered in some sequence, based on the response to the preceding item(s), and if need be a discussion with the child may be held. It is also possible to design tasks, which require many capabilities of the child to be brought into play.

Offering variations in the tasks dealing with the same concept not only provides various entry points to the children but it also allows the teacher to exactly pinpoint the problem area within that particular concept as well. A lot can be understood about the child's thinking by simply changing or reversing the order of digits in a number or by adding more digits to the number especially as far as number related concepts are concerned.



How can I increase the level of difficulty of the tasks for the children who are already able to solve the simpler ones?

Answer: Often in mathematics, particularly number knowledge, gradation can be achieved by altering the size of numbers or by slightly increasing the complexity of the processing required. Some modifications of the tasks have been indicated in the book for increasing complexity and difficulty within the descriptions provided. You can compare and modify these illustrative examples for specific grade (class) requirements, by referring to the syllabus for more specifications regarding grade-wise expected capabilities.

What should I do when a child gives an incorrect answer? How should I mark that child?

Answer: Surprisingly, an incorrect answer can offer wonderful insights into the child's thinking process. An incorrect answer does not mean that the child has not thought before answering a problem. It only means that the child uses different logic. By asking the child to explain the steps s/he has undertaken to arrive at the answer you can actually understand at which step did s/he used incorrect logic, mathematical fact or procedure and where does s/he need support.

I have forty children in my class. How will I be able to assess all of them in such a detailed manner?

Answer: First of all you do not have to assess all the students in one day. The assessment can continue over a period of time when a particular concept is going on. The whole class can be given a task and one student can be called out to have a one-on-one discussion with the teacher. The observations can be marked in the form of progress chart that have been discussed at the end of this chapter and are again referred to later in the handbook. Over a period of time then you would have assessed all your students for a particular concept.

Examples of tasks that involve the whole class at a time or engage groups of children have also been given in the source book. You could also devise your own unique ways to organise assessment for your students where you could ask the students to pair up and one student could be made responsible for recording the responses given by the other.

How would the progress chart benefit me as a teacher?

Answer: The progress chart that you will fill for each child would comprehensively convey to you the level of each child. It would be authentic recording of the child's understanding and thought process. This will help you plan your future course of action with the whole class as well as with specific students who have difficulties. This could also be shared with the child so that she becomes aware of her own understanding and recognises the areas that need more attention. Gradually the learner would become independent and take responsibility of her own learning. This document can also be shared with the parents as a record of the progress of their child.

Keeping in mind the aim of learning mathematics, how children learn and the dimensions of teaching mathematics and undertaking assessment, we present below how assessment can be made continuous, interactive, child friendly, part of the process of learning and enjoyable.

2.3 Approaching Assessment Differently

Assessment should be able to provide us with feedback as to what and how children learn. How they understand or interpret our teaching. This requires us to give them tasks that need children to use their understanding of the mathematical processes. What are the things that children are able to do? How do they interact among each other and use their knowledge to solve a problem? To answer these we need to observe, record and interpret the activities of children while they are undertaking tasks in a mathematics class and even outside.

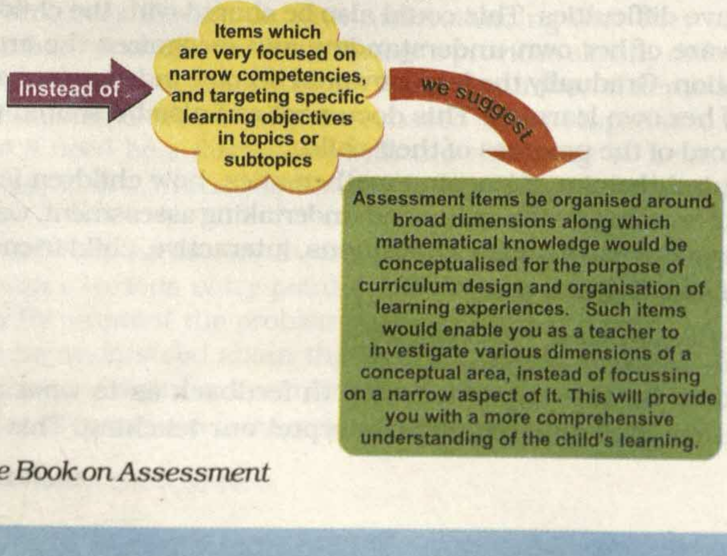
What is observed is that assessment practices are focussed on the specific objectives of the curricula. In this approach mathematical ideas and capabilities are broken down into activities of knowledge to be taught, learnt, practised and assessed. This lends itself to arriving at total marks through examinations and tests. This type of assessment provides teachers and parents with a rough idea of the mathematical achievement of the child. But this kind of assessment offers very little information to understand the 'quality' of the child's learning – what are the concepts and abilities that she is reasonably independent, where does she need support to solve problems and difficulties, what is the level and quality of her mathematical reasoning, etc.

If assessment is to benefit both the teacher and the child by directly contributing to the process and improvement of learning, then what is required is :

- Activities and tasks through which we can engage with and explore children's mathematical knowledge and reasoning. Often in carrying out these activities and tasks we provide children with opportunities not only to do what they already know, but also to try out new things and extend their knowledge.
- Observations of what children do and cannot do, and also how they go about doing and explaining their actions.
- Ability to make informed judgements based on our interpretation of observations in the light of the understanding we have on how children learn.
- More theoretical and research based understanding into the development of mathematical concepts in children.

It is possible for teachers to use assessment to explore the nature of children's learning of concepts, procedures and thinking systematically. Such an understanding of the development of mathematical knowledge as well as the status of each child's mathematical understanding would directly contribute to your ability as a teacher to plan your instruction more effectively. How can assessment be undertaken in such a way? The critical question is what kind of change is needed within classrooms and why? Some points are given below where we try to focus on the areas where a change can be brought about for a more fruitful and enjoyable teaching-learning experience.

What needs to Change and Why?



Items which are very focused on narrow competencies, and targeting specific learning objectives in topics or subtopics

Assessment items be organised around broad dimensions along which mathematical knowledge would be conceptualised for the purpose of curriculum design and organisation of learning experiences. Such items would enable you as a teacher to investigate various dimensions of a conceptual area, instead of focussing on a narrow aspect of it. This will provide you with a more comprehensive understanding of the child's learning.

→ *Instead of activities that involve only paper pencil tasks.* It is suggested that the child may also be given more of only real-life oriented activities to do and answers to report, and the teacher would use an observation sheet for notes.

→ *Instead of viewing only the final answer of the child, and hence being judgmental and product oriented.* It is suggested that assessment could be more process oriented. Ideally, even while they are attempting paper-pencil tests, children should be observed while they solve the problems, to see how they choose and alter strategies, how they respond, etc.

→ *Instead of looking for and counting only those questions that the child answered correctly.* It is suggested that as a teacher you may analyse children's errors to find out their reasoning and misconceptions in learning and take corrective steps.

→ *Instead of avoiding conversation with the whole class or group.* It is suggested that take or make opportunities to talk to children, converse and have one-on-one discussion with them. This will help you explore their thinking.

→ *Instead of using primarily group administered tests.* It is suggested that more emphasis be given to tests administered at an individual level.

→ *Instead of focussing on computation.* It is suggested that computation as one component of mathematical learning be included. Give importance to other areas of mathematical understanding such as conceptual understanding, mental representation, symbolic representation and the strategies that the learners use.

→ *Instead of having questions with 'unique' solutions or preferred method.* It is suggested that children may be allowed open ended exploration, problems with more than one right answer and also alteration in questions to permit teachers to interact with children's thinking.

→ *Instead of having activities that primarily assess children's knowledge of mathematical facts.* It is suggested that focus should be shifted from facilitating computation and mathematical reasoning to reconstruction of facts and their use in developing patterns, structures and generalisations.

→ *Instead of having activities that do not permit any discrimination between rote memorisation of algorithms.* It is suggested that the assessment offers tasks that are designed to discriminate between rote memorisation and conceptual understanding and help them develop ability to learn.

→ *Instead of focus on scattered activities of knowledge.* It is suggested that tasks to elicit more than one level of response be included.

2.4 Record Keeping and using feedback

Assessment is essentially a process of gathering information. In the assessment of learning, if marks alone are used they do not tell us much about what is being learnt or how.

The primary purpose of assessment information is to provide feedback to the child and develop more appropriate need-based teaching plans by the teacher.

This is crucial so as to enable children to develop their own confidence and to nurture emerging mathematical abilities.

In order to help the quality of learning, the following aspects may be recorded for each child:

- Mathematical concepts as per the syllabus (elaborated into various dimensions and subsections as examples.)
- Mathematical reasoning (able to follow an argument or able to provide an argument, justification, etc; able to explain how and why she did what she did and reflect on the procedures followed by others.)

Even the child's statement 'I don't know' provides us more valuable information, and we must not see it as total lack of understanding on the part of the child, and give a zero mark to the child. In addition to telling us that the child does not know the answer, it does tell us that the child is clear, confident and comfortable enough to say that she does not know.

- Attitude towards mathematics (persists at task, confident about ability to do, etc.)
- Using mathematical knowledge and techniques to solve problems (can solve problems in more than one way if possible.)
- Communication (initiates questions, shares and explains to peers, etc.)

Assessment of all the above mentioned dimensions will provide teachers with insights into the nature of individual attention to be provided during the teaching-learning process.

A progress chart given here would help you to consolidate your observations into a commentary on the child's mathematical understanding. This consolidation would be carried out periodically.

Based on the topics you have been teaching during the term and period for assessment, you would fill in the details of the conceptual area. The details of what would go into this section are provided in following chapters devoted to each of these areas by turn.

In addition, there are common features which you would need to observe regardless of the specific content. This includes aspects of mathematical reasoning, communication and attitudes.

In addition to formal notes that are kept by you, ongoing classroom observation in the form of mental notes you make as you teach and while children work, regarding aspects of each child's participation as well as individual work characteristics also form a part of assessment. Some of the aspects to note would include:

1. Does the child ask questions?
2. Is he/she able to follow arguments and make their own,
3. What does she/he do when confronted with a new type of problem, etc.

When assessment is taken across the whole class, you could also gain insights into your own pedagogic practice and identify areas where you need to focus more or alter current practices. It would also give you a sense of appropriateness of the overall level of the tasks or curriculum etc.

In order to help you use the record of progress given earlier on an illustration of assessing a child on the first conceptual area and related activity is highlighted for three children observed by a teacher. The example also focuses on how the information on assessment can be utilised for improving the proficiency level of the three children.

RECORD OF PROGRESS

Child's Name:	Recording 1 (term 1)	Recording 2 (term 2)	Recording 3 (term 3)
Conceptual Area			
Mathematical Reasoning and Metacognition			
Is able to appreciate alternative ways of solving a problem.			
Is able to invent her own new problems.			
Is able to help others at solving problems (without simply giving the answers), by explaining and illustrating.			
Is able to appreciate sources of error.			
Is able to review on her won and correct mistakes.			
Is able to use mathematical reasoning in other curricular areas.			
Mathematical Communication			
Is able to use mathematical reasoning in other curricular areas.			
Is able to explain why she did and why/how she solved the problem.			
Is able to listen to and follow explanations and solutions given by others.			
Attitudes and Dispositions			
Enjoys doing mathematics problems.			
Is confident and willing to attempt new problems.			
Is willing to persist with problems and does not give up too easily.			
Seeks out and solves new problems.			

Conceptual Area: Counting and Concept of numbers

Assessment Activity: In a given collection of objects, say marbles (upto ten in numbers), the child is asked "How many marbles are there?"

Learning Level of the Child: Class I

Assessment Procedure: Each child being assessed is asked to carry out the activity with different set of marbles and is carefully observed by the teacher. The observation for each of the child is recorded on the progress chart. An observation recorded by the teacher is given below.

Case: 1 - While counting the marbles the child touches the marbles and counts one by one (making one to one correspondence). Out of the five time of counting marbles, the child did it correctly thrice. On the remaining two occasions the child jumped a number while counting.

Proficiency level – Basic

Case: 2 - The child counts the marbles one by one (without making one to one correspondence). On all the five occasions she counted the marbles correctly.

Proficiency level – Intermediate

Case: 3 - The child is looking at the marbles and giving the correct number of marbles without one by one counting. The child is able to give correct number of marbles, as soon as they are placed before the child.

Proficiency level – Advanced

Utilising Assessment Records

Case: 1 - The child requires enrichment material. The teacher should try alternative approaches and strategies so as to provide more opportunities to the child to attain the proficiency in counting, first by using one to one counting (intermediate level) and subsequently without using one to one counting (advanced level). After the child is able to use one to one counting she may be exposed to subsequent activity, though more work needs to be done to achieve an advanced level of proficiency.

Case: 2 - Teacher should provide time and opportunity to the child so that she attains an advance level of proficiency at her own pace of learning, though she can be exposed to the next activity.

Case: 3 - The child should be considered equipped enough to go for the subsequent activity.

As a teacher you may like to design similar examples for determining the level of learning for activities related to the other conceptual areas as per syllabus. Alternative approaches and strategies as presented in the Introductory chapter may be tried out for utilising processes and outcomes of assessment for improvement of teaching-learning processes so as to bring the child to an advanced level of proficiency. It is important to understand that the level of proficiency also develops with using reasonably learnt concepts in new learning.

In addition to the record kept in the format suggested above and the notes and analysis of observations, some other material indicating the trajectory of the child's learning can also be kept. Though task based assessment is emphasised for the beginners no assessment would be complete without paper pencil tasks on worksheets. The very same worksheets that are used to introduce and practice a concept or algorithm may be used as an assessment of how quickly the child acquires the learning planned for. A careful selection from such worksheets which is analysed over a period can give very valuable insights into the ways the child learns, time she takes and conceptual routes she travels to reach the formal ability or concept. Such record keeping will also broaden the scope of assessment and make it an integral part of the learning process. One can also give dual purpose worksheets that practice a concept as well as assess the child's progress on it. Such material for assessment and analysis can also be generated from other learning, exploration and problem solving tasks that are set for a child or for a small group of children.

Often it is very useful in mathematics to introduce a new concept or procedure through setting a problem. For example a child who can handle problem solving with one step (say addition and subtraction) may be given a problem with smaller numbers that requires two steps. Care must be taken to give enough points of familiarity as well as the right kind of challenge. An analysis of how the child approaches this new problem which is not explained earlier to her can give very valuable insights of the child's independence, confidence in thinking through and in to her ability to make use for what she has learnt in learning new things. The issue here is not finding the correct answer but having a glimpse as to how the child's mind works. These special tasks sheets can be kept as a record to understand progress of the child in thinking mathematically. A non-judgemental comparative analysis of these tasks attempted by children can also provide an insight into the general way children approach such tasks and common learning routes they take. But care must be taken about what we interpret as the way children think might also influenced the way we have taught the prerequisite concepts.

The collection of this material (progress chart, teacher's analytical notes, samples of the child's work, special worksheets designed to teach as well as assess, et al) kept in a file or a portfolio can give a very good illustrated profile of the child's mathematical capabilities. It could be used to track progress, as well as to share with the parents.

2.5 Plan of the Chapter

This Source Book has six sections each discussing a conceptual area:

- Number (including number representation and decimal system)
- Number operations – addition, subtraction, multiplication and division
- Fractions
- Shapes and space
- Measurement
- Problem solving, patterns and data handling
- Each of these sections begins with an introduction discussing the key concepts and procedures, which are learnt by children. The introductory section is important as it gives an overview of the conceptual area, and indicates what to look out for as you assess the child. This will provide you with a framework within which you can interpret your observations of how children learn, progress and perform. In each conceptual area the discussion focuses on:
  - What the learning of this area includes in terms of concepts, abilities and skills, and procedural knowledge, thinking abilities, vocabulary and argumentation.
  - Indications of how one conceptual area is linked to other conceptual areas both within mathematics and across other subjects as well as in relation to the child's daily life.

- We present activities and items that can be used for assessment for different areas.
- Most of the items have a detailed discussion note. It is hoped that this will draw your attention to what to look out for when you assess, how to alter and vary the task to find out what children are doing and also questions that you can ask the child to assess their level of understanding. These notes also draw attention towards interpreting children's responses.
- The difficulty level of the items can be modified to suit each child's needs.
- Activities and items can be added to the Source Book, and modification of existing items to suit your and the child's needs is also possible.
- Several of the suggested items require different types of materials to be used. You could make some of the materials. A list of the types of materials that you can plan to collect and keep in your classroom for ready use is also highlighted in the box below.
- The last part presents indicators for the specified conceptual area and related sample tasks for assessing learning and progress of the child at three levels viz., basic, intermediate and advanced.

3

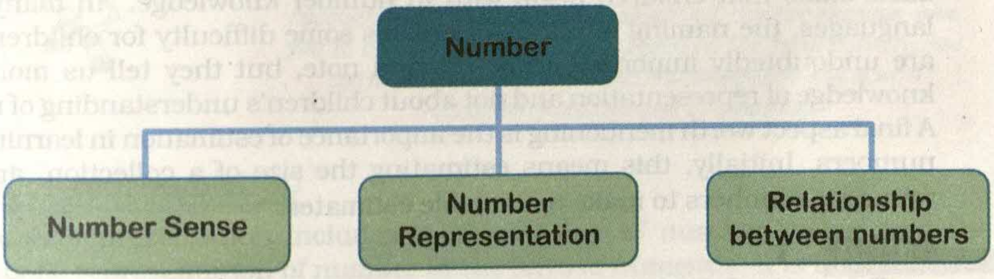
Numbers

The early stages of Mathematics learning focuses on learning about 'Number'. This consists of three inter-related but distinguishable aspects.

Number sense: This involves the ability to assess the size of a collection of objects, using estimation or counting. This is one of the most useful and basic skills in life and forms the basis of much of later learning in number.

Number representation: This involves naming and writing numbers. A deeper understanding of the concept of place value forms the basis for more abstract understanding of number and representation.

Relationships between numbers: It involves the ability to use number in ordering and sequencing. 'Before', 'after', the 'third after' etc. are some of the expressions associated with ordering. This enables children to appreciate numbers as entities in themselves, to develop an understanding of the underlying logic and the ability to play around with numbers as abstract ideas.



The development of the concept of number in children involves the development of all these three areas. Generally, in school mathematics, number representation receives the maximum attention. The children are asked to repeatedly write the same number on a page. It is important that the children learn how to represent numbers correctly, but not at the cost of development of number sense.

Mathematics textbooks suggest developing these three areas through use of concrete materials, followed by visual representations, and eventually numerical manipulations. It is important to note here, that many children who are able to answer questions that *seem* to display an understanding of number, may be answering from rote memorisation instead of real understanding. This is often not easy to distinguish, and more than one Activity may be necessary to assess whether a child has really understood the concept. For instance, most of the children can answer correctly when asked to write 3 tens + 5 ones as a numeral. This seems to indicate that they have a basic understanding of place value. But many children who can answer the above question, might falter when asked to write 4 ones + 3 tens as a numeral – showing that they were answering the first question just mechanically, without any real understanding of place value.

In the section below, these three areas are described in detail, with examples of Activities that can be used to assess the child's grasp in that conceptual area. Though the three areas have been detailed separately, the interconnectivity between them will be evident when the Activities are examined carefully.

3.1 Number sense

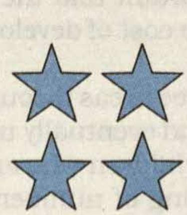
Number sense is the key conceptual part of understanding of numbers that needs to be assessed. Traditional paper-pencil testing is generally unable to capture this understanding fully, because the child can easily give the correct answer from rote memorisation without really drawing it from her 'sense of number'. It is only when we engage the child in a conversation and provide her opportunities to think aloud and share her mental processes with us that we get insights into her understanding. A variety of problems – conventional and unconventional, on number comparison, combining-decomposing, and estimation need to be asked to gauge the child's number sense.

In the early stages, the development of the naming of number and counting go together with the development of number sense. On the surface, most children maybe able to 'count' even before they enter school, but it is important to figure out whether they are mechanically counting or have they developed number sense. The tasks presented in this section elucidate how the 'number sense' of a child can be gauged.

Counting, naming of numbers, and writing numbers are some of the most basic skills that children begin with in number knowledge. In many Indian languages, the naming of numbers creates some difficulty for children. These are undoubtedly important to detect and note, but they tell us more about knowledge of representation and not about children's understanding of number. A final aspect worth mentioning is the importance of estimation in learning about numbers. Initially, this means estimating the size of a collection, and later, rounding numbers to make reasonable estimates.

ACTIVITY 1

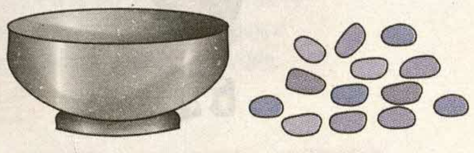
Conceptual Area: Counting and concept of number.
Material: 10 pencils, or 10 pebbles, or 10 of any small objects.
Instructions: Take a few of the pebbles, place them in a row, and ask the child. "How many pebbles?"



Note whether the child counts or just says some random number. A variation of the task can be to create a pattern using the pebbles and asking the child to recreate the same pattern exactly (he has to apply counting techniques in some form to do this task). A child with a more advanced understanding of number may be able to recognise the number of objects in a set **without actually counting** - for example, she may directly recognise that a set of stars arranged as shown here represents 4 stars, without actually counting with her fingers.

Another variation of this task could be to put some pebbles in a row and then ask the child how many are there. If she answers correctly, rearrange the pebbles to form a heap and then ask the child again, "Now how many are there?" Observe whether the child re-counts. If the child can say without re-counting that it is still the same number, ask her to explain how she knows that. This shows whether the child has understood conservation of number.

ACTIVITY 2



Conceptual Area: Counting and concept of number.

Material: About 40-50 pencils, or pebbles, or any other small objects, a large bowl.

Instructions: Tell the child: 'Put 17 counters in a bowl'. DO NOT instruct her to count and put 17 in the bowl.

Note the strategy the child employs to put 17 counters in the bowl. Rather, note ***whether*** *she counts at all, and if she does, then how she counts... in ones, or twos, or more? Depending on her confidence with the counting system, she will progress to more efficient ways of counting. Also note how she is minimising the possibility of errors in her counting/need to recount. Does she clearly put aside the counters she has counted? Does she arrange them in some order, say, in fives, or twos, to assist in tracking the counters counted and for recounting? She may make small errors, say for example, give you 16 or 18 instead of 17, but based on the total activity, you can decide how confident and knowledgeable she is with cardinality and the use of counting.*

3.2 Number representation

Number representation includes the knowledge of number names and the symbolic representation of number in the form of numerals. It is important for children to learn the oral and written representation of number in the early years. However, it is important for us as teachers to remember, that mechanically copying the number names such as 'five hundred and seventy one, five hundred and seventy two,...' are not mathematically enriching or relevant activities.

It therefore becomes essential to teach number representations in ways that employs the understanding of the child. The children could be asked to extend names to name new numbers or maybe extract information regarding number size from the name of the number. This will compel the child to construct knowledge to arrive at the answer, instead of banking upon memory.

Understanding the concept of place value is an important aspect of number representation and a core requirement of mathematics. The Activities suggested for place value help us examine some core ideas such as grouping, counting in ones, tens and hundreds.

ACTIVITY 3

Conceptual Area: Recognition of numerals and knowledge of number names.

Material: Number cards – set of cards up to a maximum of ten, with random numbers from the range of numbers you expect the child to know, including units, numbers with zero in the units or tens place, and numbers where the digits are reversed e.g. 12 and 21; 46 and 64 etc. The range of numbers represented would be based on the expected range of the child's number knowledge. Some larger numbers may also be included to explore if children know, or can extend their number knowledge range.

Instructions: Show the child one card at a time and ask 'What is this number?'

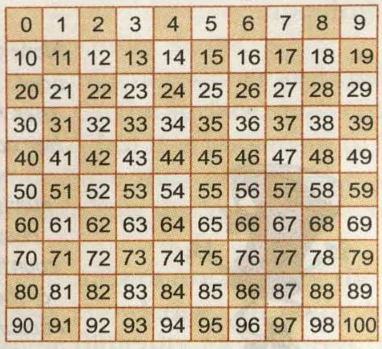
In particular, you can observe if the child is confused about naming numbers where a zero is involved, and where the numerals are reversed (64 and 46 etc). Also, note if there is a number above which the naming of numbers is confused or unsure.

You may find it interesting to ask the child to state the largest number she knows or can think of.

ACTIVITY 4

Conceptual Area: Recognition of numerals and knowledge of number names.

Material: A number board with 0-9 and 1-10 in the uppermost row, and going on till 99 or one hundred (or more) – like a snakes and ladder board.



0	1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18	19
20	21	22	23	24	25	26	27	28	29
30	31	32	33	34	35	36	37	38	39
40	41	42	43	44	45	46	47	48	49
50	51	52	53	54	55	56	57	58	59
60	61	62	63	64	65	66	67	68	69
70	71	72	73	74	75	76	77	78	79
80	81	82	83	84	85	86	87	88	89
90	91	92	93	94	95	96	97	98	100

Instructions: Point to the zero and ask the child to start reading the numbers.

This Activity will enable you to understand if the child knows number names and the counting sequence. It will not establish clearly if the child is 'reading' the numerals she is seeing or if she is simply saying things from rote memory. Note if she can recognise and name 'zero'. If she cannot and hesitates, then place a card over the zero to cover it and ask her to read from 'one onwards'. See how far she is able to read.

If the child succeeds in the above part, then you can try this: point to a number midway, in the range the child is able to count and ask her to start counting from that number onwards. This will enable you to judge if her earlier response was purely rote recitation or if she was reading the numbers with recognition.

You could also pick any number, say 12, and then run your finger down the column to '22', '32' etc., and ask the child to name these numbers. If the child has a good sense of place value and the structure of the number naming system, she will be able to name subsequent numbers with ease.

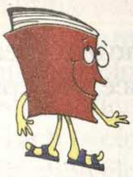
ACTIVITY 5

Conceptual Area: Writing numerals,

Material: Number cards — set of cards up to a maximum of 10, with random numbers from the range of numbers you expect the child to know, including units, numbers with zero in the units or tens place and numbers where the digits are reversed e.g. 12, 21; 46, 64 etc. The range of numbers represented would be based on the expected range of the child's number knowledge. Some larger numbers may also be included to explore if children know and can extend their number knowledge range.

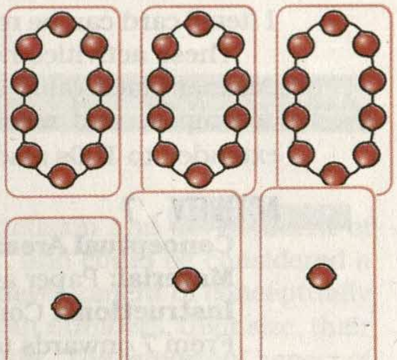
Instructions: Pick out numbers beginning with smaller numbers with which the child may be more familiar, and then at random.

Read out the number on the card and ask the child to write the numeral on the paper or slate provided.



As in the case of reading numbers, here too note if the child is confused about numbers where a zero is involved, and where the numerals are reversed. Also note if there is a number above which the writing of numbers is confused or unsure.

ACTIVITY 6



Conceptual Area: Knowledge of grouping round tens and ones (basis for decimal representation).

Material: Bundles of sticks depicting sets of ten (10 sticks tied together) and loose sticks depicting ones.

or

Unit Ten Hundred (UTH) cards with beads depicting sets of ten (ten cards) and cards depicting one bead (ten cards).

or

UTH kit, with single cubes (ten) and tens bars (ten).

Instructions (*To familiarise the child with the cards*): Show all the ten bead cards to the child and say: 'Each of these cards has 10 beads on it'. Show the single bead card and ask: 'How many beads are there in each of these? Put before the child one 10-bead card and ask: 'How much is this?' Pick that up and freshly lay two 10-bead cards and ask the same question.

(a) Pick up all the cards and out of them pick out one 10s and three 1s and lay them out in front of the child. Ask: 'How much is this?'
Pick up all the cards and out of them pick out three 10s and five 1s and lay them out in front of the child. Ask : 'How much is this?'
Pick up all the cards and out of them pick out two 10s and seven 1s and lay them out in front of the child. Ask: 'How much is this?'

Note the child's response in each case. Also, observe and note down whether the answer is provided immediately, whether she physically picks up or considers the tens and the ones and computes and announces the answer (also note if she proceeds with the tens first and then the units or the other way round), and also note if she counts all. This will indicate to you the familiarity and confidence the child has with UT and handling numbers in the form of units and tens.

(b) Give all the cards to the child and ask her to:
'Represent 12 with these cards'.
'Represent 40'.
'Represent 23'.

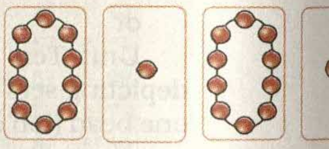
Note the child's response in each case. Is she just counting a **total** of 12 cards (without looking at what those cards stand for), instead of actually choosing tens and ones cards to represent 12? If she is proceeding correctly, note whether the answer is provided by immediately laying out the cards working with tens and then with ones, or the other way around.

(c) More challenging tasks can be given to children who are already thorough in the tens-ones grouping and counting. For instance, you could give them all the cards, and say:
'Represent 20 using exactly 11 cards'.

If the child is able to do this, it means that she has internalised the fact that 1 'tens' card can be replaced with 10 'ones' cards.

These activities help to assess both readiness for and familiarity with the decimal place value system, by basically investigating if children are able to decompose and recompose number in tens and units. The same could be extended to 100s also.

ACTIVITY 7

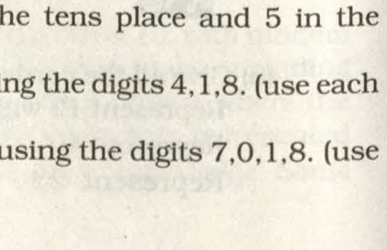


Conceptual Area: Number Sense and representation.
Material: Paper and pencil.
Instructions: Continue the number series:
From 7 onwards to five more numbers, 7,___,___,___,___,___.
From 28 onwards to five more numbers. 28,___,___,___,___,___.

Note in particular the child's facility while going over from 9 to 10 and from 29 to 30. In case the child hesitates, you may want to prompt and scaffold so that the child is able to go over and continue the series. It is an indication that the child is aware and expects some change in the writing pattern, but is unsure about the change itself. More familiarity in the form of exposure and opportunities to count over is most probably what she needs.

ACTIVITY 8

Conceptual Area: Place value system.
Material: UTH Blocks.
Instructions: Using UTH blocks represent 100, 137, 103, 230 etc.



ACTIVITY 9

Conceptual Area: Place value system.
Material: Paper and Pencil.
Instructions:

(a) Name a 4-digit number which has 7 in the tens place and 5 in the thousands place.
(b) Make the largest 3-digit number possible using the digits 4,1,8. (use each digit only once).
(c) Make the smallest 4-digit number possible using the digits 7,0,1,8. (use each digit only once).

ACTIVITY 10

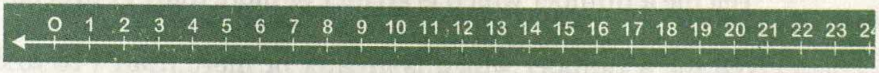
Conceptual Area: Number Representation on number line.

Material: Paper and Pencil.

Instructions: Tasks involving the number line could be used with children who do have some exposure and familiarity with the number line. Here we would be interested to know if children are able to visualise relative positions of numbers on the line.

Draw a number line beginning with zero and continuing till beyond 50. Ask the child to point out some numbers for you on the line—5, 32, 43 etc. Once the child starts identifying the numbers correctly, ask her questions like:

'Which is closer to 15 : 32 or 41?'
'Which is closer to 15 : 11 or 19?'
'Which is closer to 23 : 19 or 29?'
'Which number is equidistant from 15 and 19?'



3.3 Number Relationships

In many respects, number relationships is an extension and development of what we have described earlier as 'number sense', and could be considered a more advanced level of the same. It involves the development of conceptually more powerful and meaningful ways of thinking about numbers, their size, their relationship with each other, and their representation. The important aspect of this section is that number is dealt with in a more 'abstract' manner, rather than being context bound. This portion will deal with number operations but would focus more on the concept of number rather than computation. Several aspects of this are also taken up in the later subsection called 'patterns'. We have not separately emphasised aspects of number learning such as 'odd-even', 'prime', 'factor', 'multiple' in this Source Book because it is important to see these in relation to each other rather than viewing them in isolation. The interconnections facilitate both generalisation and abstraction. Studying all of them together enables us to see the patterns involved and arrive at certain generalisations.

ACTIVITY 11

Conceptual Area: Ordering of numbers.

Material: Number cards — set of cards up to a maximum of ten, with random numbers from the range of numbers you expect the child to know, including units, numbers with zero in the units or tens place and numbers where the digits are reversed e.g. 12, 21; 46, 64 etc.

Instructions: Place all the cards out in front of the child and ask: 'Which is the greatest or smallest of these numbers?'

Place all the cards out in front of the child. Pull out one card (about the middle-most number) and place it above the set. Ask the child: 'Put all those numbers which are smaller than this on this side (indicating the left) and all those numbers that are greater than this on this side (indicating the right).

Note whether the child scans all the cards and picks out the greatest or the smallest, also note if she chooses a card with a number which is greater or smaller, but may not be the greatest or the smallest. You could ask her to check (whether she is right or wrong) and note what strategy she uses to check her answer. You would like to note whether she compares numbers and then reviews. Also check out - is she mixing up, say '21 with 12' (digit reversal) or, does she consistently misread numerals where there is a zero?

ACTIVITY 12

Conceptual Area: Ordering of numbers.

Material: Oral task (or paper and pencil).

Instructions: Ask the child:

'Tell me a number which is greater or more than 11 and smaller or less than 20'.

'Tell me a number which is greater or more than 110 and smaller or less than 120.'

'Which of the following numbers are between 155 and 165?'

160, 150, 145, 166, 156, 153, 162

ACTIVITY 13

Conceptual Area: Recognising number patterns.

Material: Paper and Pencil.

Instructions: Continue the series:

8, 10, 12, ___, ___, ___, ___, ___.

13, 20, 27, ___, ___, ___, ___, ___.

100, 117, 134, ___, ___, ___, ___, ___.

Note, that these tasks are more difficult than simply continuing a number sequence, and can be used once children are able to do simple sequencing tasks comfortably.

ACTIVITY 14

Conceptual Area: Ordering of numbers.

Material: Paper and pencil.

Instructions: Given five numbers: 1007, 1070, 1700, 7100, 7001, arrange them from the greatest to the smallest.

3.4 Indicators

The indicators of quality for assessment of children's learning and progress in development of numbers for Classes I-V are presented in the following table. Against each conceptual area, three broad levels of proficiency are described. The descriptors given for each proficiency level describe what the child should be able to do if she is currently functioning at that level of proficiency. The aim of assessment is to see whether the child is moving to higher levels of proficiency in that conceptual area over time.

It is important to note that these levels are just a sample of the kind of progress chart that the teacher can use to track the progress of a child over time. The teachers can develop their own indicators based on the syllabus.

	Conceptual area	Sample Activities	Proficiency Level		
			Basic	Intermediate	Advanced
Number Sense (3.1)	Counting and concept of number.	1, 2	Occasionally uses techniques (like 1-1 correspondence through finger counting) correctly to count the given objects*(Sample Activities :1, 2).*	Mostly able to count the given number of objects correctly using correct techniques*(Sample Activities: 1, 2).* Sometimes uses grouping techniques and strategies to count larger sets of objects *(Sample Activity: 2).*	Consistently able to count the given number of objects correctly using correct techniques*(Sample Activities:1, 2).* Demonstrates conservation of number and advanced understanding of number*(Sample Activity:1).* Consistently uses effective grouping techniques and strategies to count larger sets of objects*(Sample Activity: 2).*
Number representation (3.2)	Recognising/ writing numerals and knowledge of number names.	3, 4, 5	Occasionally able to read a number (from among the numbers expected to be known) correctly when it is written in numerals *(Sample Activities:3, 4).* Occasionally able to write a number correctly when it is spoken*(Sample Activity: 5).*	Mostly able to read a number (from among the numbers expected to be known) correctly when it is written in numerals *(Sample Activities:3, 4).* Mostly able to write a number correctly when it is spoken*(Sample Activity: 5).*	Consistently able to read a number (from among the numbers expected to be known) correctly when it is written. This includes numbers with 0, 'reversed' numbers (example: not reading 34 as 43) etc.*(Sample Activities:3, 4).* Consistently able to write numbers - without confusing numbers with 0, 'reversed' numbers etc.*(Sample Activity: 5).* Able to extend the understanding of the naming system to name larger numbers (more than what she is currently expected to know) in different contexts, including in real life situations.
	Understanding place value system (grouping round tens and ones).	6, 8, 9	Occasionally able to use tens-ones grouping to represent numbers - using UTH blocks or other models. Occasionally able to compose or decompose numbers using tens- ones grouping*(Sample Activities: 6,8).* Occasionally able to identify place value of a given digit in a number, but may need a lot of support to solve problems based on place value*(Sample Activity: 9).*	Mostly able to use tens-ones grouping to represent numbers using UTH blocks or other models. Mostly able to compose or decompose numbers using tens- ones grouping*(Sample Activities: 6,8).* Mostly able to identify place value of a given digit in a number, and do simple tasks based on the idea of place value *(Sample Activity : 9).*	Consistently able to use tens-ones grouping to represent numbers using UTH blocks or other models. Consistently able to compose, or decompose numbers using tens-ones grouping, even alternate compositions like 21 as 1 ten + 11 ones*(Sample Activities: 6, 8).* Able to successfully handle challenging tasks based on the idea of place value *(Sample Activity: 9).* Uses tens-ones based composing and decomposing (on her own) to solve problems in the classroom or in real life situations - for instance, breaking up 35 to 30 + 5 mentally while computing 70 - 35.

	Conceptual area	Sample Activities	Proficiency Level		
			Basic	Intermediate	Advanced
Number relationships (3.3)	Knowing numbers in sequence (including number line).	7, 10	Occasionally able to say the number that comes after/before a given number, or between two given numbers. Occasionally able to perform simple sequence completion tasks *(Sample Activity: 7).*	Mostly able to say the number that comes after/ before a given number, or between two given numbers. Mostly able to perform simple sequence completion tasks *(Sample Activity: 7).* Mostly able to reason about relative distance and position of numbers on the number line*(Sample Activity: 10).*	Consistently able to say the number that comes after/before a given number, or between two given numbers. Consistently able to perform sequence completion tasks, including complex ones, including ones where they have to go over from say, 29 to 30 etc.*(Sample Activity: 7).* Consistently able to reason about relative distance and position of numbers on the number line *(Sample Activity: 10).*
	Ordering numbers.	11, 12, 14	Occasionally able to tell a number between two given numbers (*Sample Activity : 11).* Occasionally able to compare the number of objects in two given sets to say which is more/ less. Occasionally able to identify the greater/ smaller of two given numbers *(Sample Activities: 12, 14).*	Mostly able to tell a number between two given numbers (*Sample Activity : 11).* Mostly able to compare the number of objects in two given sets to say which is more/less. Mostly able to identify the greater/smaller of two given numbers *(Sample Activities: 12, 14).*	Connsistently able to tell all the numbers between two given numbers (*Sample Activity : 11)* Consistently able to compare the number of objects in two given sets to say which is more/less, even in complex cases where the sets may contain different objects. Consistently able to identify the greater/smaller of two given numbers and able to answer more complex comparison tasks.*(Sample Activities: 12, 14, including harder cases).* Uses ordering of numbers for comparing quantities in real life situations – like comparing the prices at two shops, etc.
	Recognising number patterns	13	Occasionally able to recognise number patterns, may need support to identify the pattern. *(Sample Activity: 13).*	Mostly able to recognise number patterns on her own. *(Sample Activity: 13).*	Consistently able to recognise number patterns, including complex ones. *(Sample Activity: 13).* Looks for patterns (on her own) in situations and is able to generalise and explain patterns observed.

4

Number Operations

Addition, Subtraction, Multiplication and Division

Number operations are a continuation of number relationships that was discussed in the preceding section. Daily life situations of children provide meaningful contexts for them to learn number operations. Word problems provide this meaningful context to the children and offer them an opportunity to construct number operations. Children encounter numerous mathematical problems everyday, some of which they may be able to solve while the others may be beyond their level of understanding. Therefore in order to assess children it is important for us to understand the increasing level of complexity of word problems. This will enable you to select problems that are appropriate to the child's level of understanding. This will also enable you to understand which problems can the child solve independently and where does she need additional support.

The aspects that need to be included while assessing a child's understanding of number operations are:

- **Relating operations to situations:** This includes the child's representational capabilities – from concrete situations to numerical representations in terms of operations and conventional signs.

  The child's ability to formulate problems of different types to fit in a given number sentence is also a part of the child's ability to relate operations to situations. For example, the teacher could ask the child to create a problem for the number sentence 2 + 5 = 7.
- **Fluency in computation**: The child's ability to solve graded problems of different types – starting from the ones that are conceptually simple, to the difficult.

  Understanding and application of properties (like commutativity of addition or multiplication) and shortcuts in computation can also be assessed as a part of the child's computational ability.
- **Using problem solving strategies:** The efficacy and range of procedures used by the child to solve problems.

The operations will be discussed under two sections – Addition and Subtraction, and Multiplication and Division.

4.1 Addition and Subtraction

We have dealt with addition and subtraction together because these two are complementary operations. There is no conceptual difference in these two operations. Teaching and assessing these two operations together enables the teacher to understand the specific area where the child faces problems. In the Source Book we have provided tasks that deal with both these operations together. More such tasks can be developed for both assessment and teaching.

4.1.1 Relating operations to situations (Addition and Subtraction)

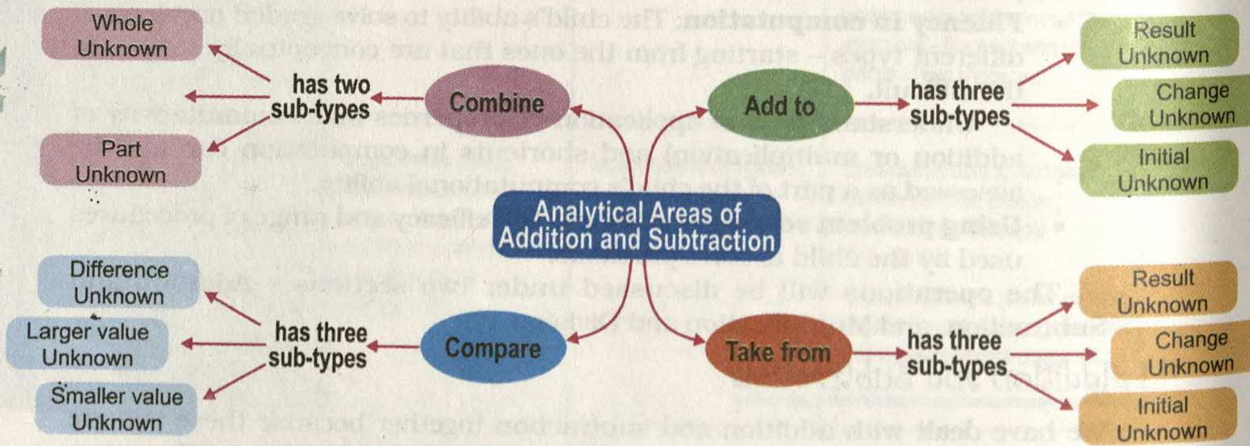
We start with a word on the importance of problems in learning about any operation. A variety of situations exist in real life where computations are needed. Children need to learn about operations because of this very reason. But often, this gets reversed in the school situation – where children learn to perform computations, and then are given situational problems to gain fluency in those operations!

In actuality, children can really understand the meaning of an operation only through contextual problems. So, what are the different type of problems that the child needs exposure to in Addition and Subtraction? It often happens that while dealing with Addition-Subtraction, usually just one or two types of situations are taken up – "Tarun had 4 bananas, Ratna had 3. How many did they have altogether?" or, "Seema had 8 sweets. She ate 3 of them. How many sweets are left?"

But there are actually a number of different types of Addition and Subtraction situations that the child needs to be familiar with. Four main analytical categories for types of word problems in addition and subtraction are:

- Join sets - 'Join' problems are based on situations that involve increasing a quantity – that is, there is an initial quantity, a change, and the resulting quantity.
- Combine (Part-Whole problems) - 'Combine' problems are based on situations that involve combining two groups of things.
- Separate sets - 'Separate' situations are the opposite of join problems – where the initial quantity is being decreased by an amount of change to get the resulting quantity.
- Compare - 'Compare' problems involve the comparison of two sets or quantities.

The types and subtypes of problems mentioned are for the teacher to gain clarity on the different types and subtypes of the problems that exist under each concept. It is not important for the child to know this terminology.



Illustrative examples

Category	Problem type	Sample Problems and Representations
Join	Result unknown	Rani had 5 pencils. Her father gave her 2 more. How many does she have now?
Join	Change unknown	Rani had 5 pencils. Her father gave her some more. Now she has 7 pencils. How many pencils did her father give her?
Join	Initial unknown	Rani had some pencils. Her father gave her 2 more. Now she has 7 pencils. How many pencils did she have to start with?
Combine	Whole unknown	Lata has 5 kites and Rama has 2 kites. How many do they have altogether?
Combine	Part unknown	Lata and Rama together have 7 kites. Lata has 5 kites. How many kites does Rama have?
Separate	Result unknown	Ali had 7 mangoes. He ate 5 of them. How many mangoes does he have now?
Separate	Change unknown	Ali had 7 mangoes. He ate some of them. Now he has 2 mangoes left. How many mangoes did he eat?
Separate	Initial unknown	Ali had some mangoes. He ate 5 of them. Now there are 2 mangoes left. How many mangoes did he have to start with?
Compare	Difference unknown	Ali has 7 kites. Rani has 5. How many more kites does Ali have than Rani? Or Ali has 7 kites. Rani has 5. How many less kites does Rani have than Ali?
Compare	Larger value unknown	Ali has 5 kites. Rani has 2 kites more than him. How many kites does Rani have? Or Ali has 5 kites. He has 2 kites less than Rani. How many kites does Rani have?
Compare	Smaller value unknown	Ali has 7 kites. Rani has 2 kites less than him. How many kites does Rani have? Or Ali has 7 kites. He has 2 kites more than Rani. How many kites does Rani have?

It is important for the child to be presented with a number of these situations, where the purpose is for her to decide which operation to use. More illustrative Activities are provided here.

ACTIVITY 15

Conceptual area: Relating operations to situations
Material: Counters or pebbles and two bowls.
Problem type: Combine, Whole unknown.
Instructions: Place the pre-arranged bowls before the child and say: 'There are 11 counters in this bowl and 5 in that bowl. In all how many counters are there?'

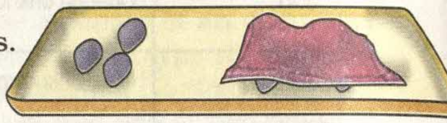
ACTIVITY 16

Conceptual area: Relating operations to situations.
Material: Red and blue counters and 2 bowls.
Instructions: Place 7 blue counters in one bowl and 5 red counters in the other bowl. Provide the child with more of red and blue counters.

Ask the child: "Make the counters in both the bowls equal."

Note whether the child is able to judge which bowl has more counters and how she makes sure that the number of counters are made equal.

ACTIVITY 17



Conceptual area: Relating operations to situations.

Material: Pebbles and a handkerchief.

Problem type: Combine, part unknown.

Instructions: Present a tray to the child in which there are 3 pebbles visible and 5 more are hidden under a handkerchief.

Ask her: 'In all there are 8 pebbles. How many are there under the handkerchief?'

For this task, initially you can start with smaller numbers, and as the child demonstrates a better understanding, go on to larger numbers – like 15 visible and 7 hidden, etc.

Step Up or Step Down!

The problem types that have been given in the following section have been arranged in the increasing order of complexity. This helps vary levels based on a child's capacity. For instance, if a child is unable to solve a simpler task, in which the 'whole' is unknown, then she may be given task where the change is unknown. This will help you determine the exact problem that the child faces and help you plan your teaching accordingly.

ACTIVITY 18

Conceptual area: Relating operations to situations.

Material: Paper and Pencil or oral task.

Instructions: Provide problems of increasing complexity levels like the following.

Problem type: Join, Result Unknown.

(a) Faiz ate 6 laddus on the festival of Id. He was still hungry so he ate 4 laddus more. How many laddus did Faiz eat?

Problem type: Join, Change Unknown.

(b) Suma had 8 story books and she got a few more on her birthday. Now she had 12 books. How many books did she get on her birthday?

Problem type: Join, Initial Unknown.

(c) Sushma had some kites. She found 2 more kites on her terrace. Now she had 11 kites. How many kites did Sushma have to start with?

ACTIVITY 19

Conceptual area: Relating operations to situations.

Material: Paper and pencil.

Instructions: Provide problems of increasing complexity levels like the following.

Problem type: Separate, Result Unknown.

(a) 6 children were playing in a garden. 3 of them went away after some time. How many children remained in the garden?

Problem type: Separate, Change Unknown.

(b) A dog was given 7 rotis. He ate some of them. 3 rotis were left. How many rotis did the dog eat?

Problem type: Separate, Initial Unknown.

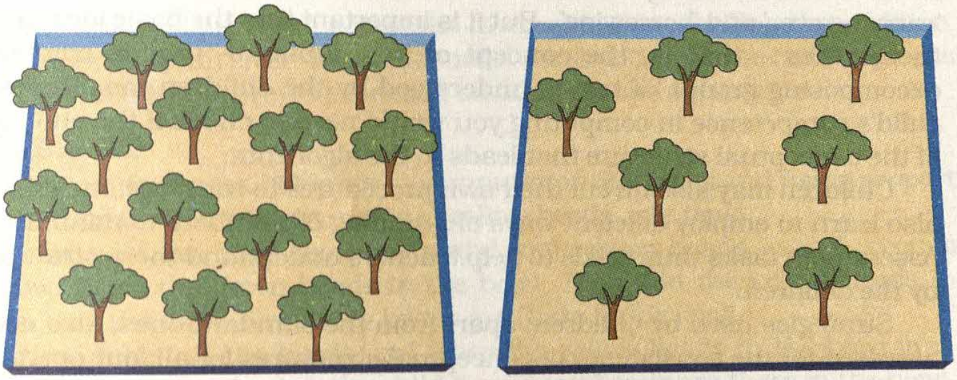
(c) A tree had some apples. Some children plucked 7 apples. Now there were only 11 left on the tree. How many apples were there on the tree in the beginning?

ACTIVITY 20

Conceptual area: Relating operations to situations.

Material: Paper and pencil.

Instructions: Provide problems of increasing complexity levels like the following.



Problem type: Compare, Difference Unknown.

(a) In one orchard there are 15 orange trees and in the second orchard there are 7 orange trees. So how many more trees does the first orchard have?

Problem type: Compare, Larger Value Unknown.

(b) Anwar's house has 12 windows. Tarun's house has 2 windows more than Anwar's house. How many windows does Tarun's house have?

(c) Anwar is 12 years old. He is 2 years younger than Ravi. How old is Anwar?

Problem type: Compare, Smaller Value Unknown.

(d) Zoya climbs 53 stairs in one minute. Mohan climbs 5 stairs less than Zoya in one minute. How many stairs does Mohan climb in one minute?

(e) Zoya is 15 years old. She is 5 years older than Mohan. How old is Mohan?

ACTIVITY 21

Conceptual area: Relating operations to situations Addition, Subtraction

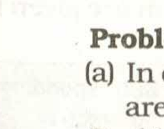
Material: Paper and pencil

Instructions: Provide an addition or subtraction fact to the child and ask her to make one or more problems that match it. For instance, ask the child:

(a) Make up your own problem for 3 + 7 = ?

(b) Make up your own problem for 8 - 3 = ?

(c) Make up your own problem for 7 + ? = 22



Most kids will initially come up with the 'Result Unknown' type situation for the first two cases. Challenge the children who are already at an advanced level to come up with different types of situations (as per the categories in the tables), instead of just the usual type.

4.1.2 Fluency in computation (Addition and Subtraction)

Computation in the basic operations involves development of informal strategies, ability to estimate, and also the more abstract understanding of the operation and the properties of the operation.

Computation involves the child's ability to

- develop informal strategies,
- estimate,
- abstract the operation, and
- abstract the properties of the given operation.

Children use a variety of strategies like 'counting all', 'take away' etc. to solve the problems given to them. They also need to master the standard procedures based on the concept of place value to add or subtract large numbers. This involves remembering a number of rules related to what is normally called 'carrying over' and 'borrowing'. But it is important that the basic idea underlying these rules – that is, the concept of 'regrouping' – that is composing or decomposing groups of ten, is understood by the children too. To assess the child's competence in computing you would need to check if the child is aware of the conceptual structure that leads to the algorithm.

Children may also invent their own procedures to compute. In fact, children also learn to employ efficient ways of grouping of numbers to minimise errors. Assessment tasks thus needs to help teachers understand these strategies used by the children.

Strategies used by children, apart from the standard ones, also need to be assessed for the consistency of conceptual structures for all four operations.

Some strategies that children use for Addition and Subtraction are given in the table below.

Procedure	**Additive Procedures Description**	**Corresponding Subtractive Procedures**
Count all	The child physically puts the two part quantities together using objects or pictures and then counts the whole collection.	Take away
Add on up to	Similar to the above, except that the child counts out the first quantitythen continues to add on the second.	Separate to
Count on	The child does not explicitly count out the first quantity, but and starts from that number or the next to count on by the second quantity. It can be noted here whether the child demonstrates an understanding of commutatively by choosing the more efficient way of computing – that is, choosing to start with 5 (the bigger number) and count on 2 (the smaller) rather than the other way around.	Count down
Count up to	This procedure, though additive in nature, is actually used for subtraction situations and demonstrates the child's understanding of the relation between addition and subtraction In this, the child when faced with a subtraction problem of the type 7 – 2, counts up to 7 (the bigger number) - starting with 2 (the smaller number).	Count down to
Use known facts	A child at this level of thinking already knows some basic facts and uses those to arrive at the unknown sum. For instance, to find 8 + 3, the child might use 'up over tens', making use of 8 + 2 = 10, a fact known to her (8 + 3 = 8 + 2 + 1 = 10 + 1 = 11) Decomposition: the child breaks the numbers into convenient numbers that she can handle. For example, 15 + 17 = 10 + 5 + 10 + 7= 20 + 10 + 2= 32. The child can handle such calculations mentally without using paper and pencil.	Use known facts

ACTIVITY 22

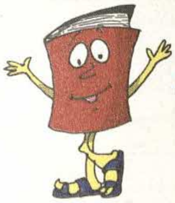
Conceptual area: Fluency in computation (Knowledge of basic facts).

Material: Paper and pencil.

Instructions: Give problems based on basic addition (later subtraction) facts.

Add: 4 + 5 = ___

Add: 7 + 8= ___



Many such activities are needed to assess the child's level of comfort with basic addition facts. Noting down the strategies that children use for calculating will help you assess the proficiency level of your children.

The child might make a mistake in counting or recalling the addition fact and hence make an error. You could model the situation for the child appropriately as described in the previous section on addition and subtraction.

ACTIVITY 23

Conceptual area: Fluency in computation (Knowing and using properties).

Material : Some green, red and yellow beads, one bowl.

Instructions: Take some green and some yellow beads – say 4 green and 2 yellow. Place the green beads in the bowl. Then add the yellow beads in the bowl. Ask the child how many beads are there in the bowl.

Now, take out the beads and place the yellow beads in the bowl first and then the green beads. Ask the child how many beads are there in the bowl.

Observe whether the child is counting the beads again in the second problem or whether the child uses the previous answer.

ACTIVITY 24

Conceptual area: Fluency in computation (Knowing and using properties).

Material: Some green, red and yellow beads, one bowl.

Instructions: Take some beads of each colour – say 4 green, 3 yellow and 2 red. Place the green beads in the bowl. Then add the yellow beads in the bowl. Now add the red beads. Ask the child how many beads are there in the bowl.

Change the order of placing the beads in the bowl (in the order red, yellow, and green) and ask the child the total number of beads in the bowl now.

Observe whether the child knows that the total number of beads in the bowl remain the same after reversing the order of placing the beads.

ACTIVITY 25

Conceptual area: Fluency in computation (Understanding and using standard algorithms)

Material: Paper and pencil

Instructions:

(a) 13 + 4 =

(b) 27 + 5 =

(c) 12 + 13 =

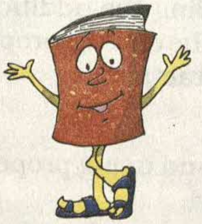
(d) 20 + 13 =

(e) 35 + 47 =

Sample solution 1 (incorrect):

(a)

	1	3	
+	4		
	5	3	



Note if the child makes mistakes in the vertical alignment of the numbers. In that case, she may not be clear with the concept of place value.

Sample solutions 2 and 3 (incorrect) and analysis:

(b)

	3	5	
+	4	7	
	7	2	

Here the child has 'carried over' 1, but probably forgotten to add it.

(c)

	1		
	3	5	
+	4	7	
7	1	2	

Analysis: The child has not understood that the '1' in 12 represents 1 tens, and hence places it this way. She is not fully clear about the concept of 'regrouping' while following the standard procedure for addition.

The concept of place value is of utmost importance while carrying out most standard operations. If the child has learnt the procedure mechanically, without the concept of 'regrouping' ones to carry over to tens and so on, she will make mistakes as in Sample solutions 1 and 2. If not cleared, this kind of errors can persist even when the child is adding 3-digit numbers.

Other possible errors could be, writing 20 + 13 as 30, by taking 0 + 3 to be 0. While doing larger problems also, numbers with 0 in different places need to be included to check if any misconceptions related to addition with 0 remain.

Similarly, graded (a to e) problems can be used at different levels in the case of subtraction also. In subtraction too, special attention has to be given to cases with 0, especially when there are 1 or more zeroes in the number from which the child is asked to subtract.

ACTIVITY 26

Conceptual area: Fluency in computation.

Material: Paper and pencil.

Instructions: Ask the child to analyse the following problems and write which ones are done correctly and which are incorrect, and why.

(a)
$$\begin{array}{r} 20 \\ -\ \ 7 \\ \hline 27 \end{array}$$

(b)
$$\begin{array}{r} 72 \\ -26 \\ \hline 46 \end{array}$$

(c)
$$\begin{array}{r} 105 \\ -\ \ 58 \\ \hline 57 \end{array}$$

Discussion

(a) The child, perhaps, has difficulties with regrouping and simply subtracts the smaller number from the larger (0 from 7).

(b) Is correct.

(c) The child has regrouped 1 hundred, and then 1 ten correctly to get 15 – 8 as 7. But then, she did not keep track of the 1 ten that was transferred to the units place.

Prompt your children to solve the sums and find out themselves before deciding. This will provide you a chance to see the strategies that the child is employing and also the related conceptual structures.

As the children compute, observe what strategy the child is using. Note whether she is

(a) picking up and using the counters (or any physical objects to count).

(b) writing on a piece of paper and computing .

(c) using some other strategy - like computing mentally or using shortcuts. For example, solving 40 – 18 mentally, by first doing 40 – 10 = 30, and then 30 – 8 = 22.

This will give you an indication of her confidence with numbers and computing. The child must progressively move from using physical aids to computing without aids, using a range of strategies.

ACTIVITY 27

Conceptual area: Fluency in computation (Knowing and using properties).

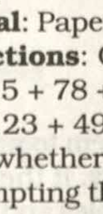
Material: Paper and pencil.

Instructions: Give children problems like these:

(a) 15 + 78 + 5

(b) 123 + 496 + 7

Check whether they are able to use properties and shortcuts independently while attempting these problems, without any prompting.



Observe whether the child knows that the total number of beads in the bowl remain the same after reversing the order of placing the beads.

Children who have reached an advanced level should use various strategies and properties like these to compute more efficiently in any situation.

4.2 Multiplication and Division

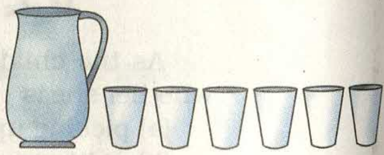
3.2.1 Relating operations to situations (Multiplication-Division)

In multiplication and division the main problem types are:

Operation	Problem type	Sample Problems and Representations
Multiplication	Multiplying factor	Chunni has three flowers and Munni has three times as many flowers. How many does Munni have?
	Rate	There are seven children and each one has two balls. How many balls are there in all?
	Cartesian Product	A dress is made in three different sizes and four different colours. How many dresses do I have from which to choose?
Division	Sharing	There are 15 watermelons. Subba wants to arrange them in 3 equal rows. How many should he put in each row?
	Grouping	There are 15 watermelons. Subba wants to arrange 5 in each row. How many rows can he make?

The types and subtypes of problems mentioned are for the teacher to gain clarity on the different types and subtypes of the problems that exist under each concept. It is not important for the child to know this terminology.

ACTIVITY 28



Conceptual area: Relating operations to situations.
Material: Water - six tumblers full.
Problem type: Division as equal sharing.
Instructions

(a) There are 6 glasses of water in the container and it has to be given to 3 children equally. How many glasses of water would each get?

At first the child can be asked to actually measure and find the answer.

Later as the problem is posed again and again, they could be prompted to generalise and answer verbally.

(b) The teacher has 8 notebooks. The notebooks have to be divided equally between 3 children. How many notebooks would each child get? How many would be left over?

ACTIVITY 29

Conceptual area: Relating operations to situations.
Material: Some pebbles.
Instructions

(a) Call 3 children and give them 15 pebbles. Ask them to predict how many would each get if the pebbles were divided equally. If they are not able to do it mentally, ask them to actually divide the pebbles and give the answer.

(b) Now extend the task by asking them to predict how many each one would get if they were given 18 pebbles to share.

(c) Convert the problem into a multiplication one by asking them that, if each wanted 5 pebbles how many pebbles should be there in all.

At this level, understanding of division situations where remainder is involved can also be introduced.

(d) Call 3 children and give them 17 pebbles. Ask them to predict how many each would get if the pebbles were divided equally. Check how they deal with the 'remaining' pebbles – can they make out 2 pebbles will be left, etc.

ACTIVITY 30

Conceptual area: Relating operations to situations

Material: Place about 50 counters and provide child with loops of string/ chalk and a large board to draw on.

Instructions: Ask the child to model different multiplication and division situations. For instance:

Show me the following with the materials.

If 24 sweets are distributed equally in three groups then each group would get 8 sweets.

If 24 sweets are distributed equally in a group of 3 then 8 groups will be formed.

You have 24 sweets. If you start removing sweets in groups of 3 each, you can remove 8 times.

8 times 3 is 24.

3 times 8 is 24.

ACTIVITY 31

Conceptual area: Relating operations to situations.

Material: Paper and pencil.

Problem type; Multiplication as rate

Instructions

(a) There are 6 biscuits in a packet. How many biscuits are there in 4 such packets?

(b) Tarun's teacher asks him to do 5 Math problems at home daily. At this rate, how many problems would he have to do in the month of June?

ACTIVITY 32

Conceptual area: Relating operations to situations.

Material: Paper and pencil.

Instructions

Match the operations in group A with the appropriate situations in group B.

Group A	Group B
10 ÷ 2= ?	● Ninety bottles had to be placed equally in two rows. How many bottles were there in each row?
10 x 2 = ?	● Mother bought ten sweets and shared them equally between Munni and Chunni.
90 ÷ 2 = ?	● Ten mothers came with two children each. How many children were there in all?
90 - 2 = ?	● Out of ninety bottles two bottles broke. How many were left?

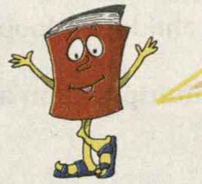
ACTIVITY 33

Conceptual area: Relating operations to situations.

Material: Paper and pencil.

Problem type: Multiplication as Cartesian product.

Instructions: A balloon seller has balloons of 5 different shapes. Each shape is available in 3 different colours. How many different types of balloons can a child choose from?



The child who may not be able to model the situation mentally may be encouraged to draw the balloons in different colours. At times children might use a hit and trial method to arrive at the answer. In such a case she may need more scaffolding from the teacher to help her model the situation.

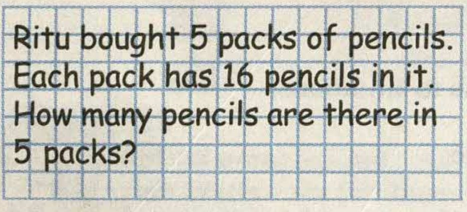
ACTIVITY 34

Conceptual area: Relating operations to situations.

Material: Paper and pencil.

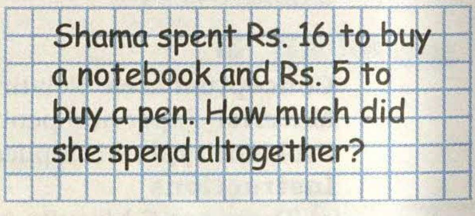
Instructions: Make up a problem which corresponds to the multiplication, '16 × 5' in the context of shopping.

Sample solution 1:



Ritu bought 5 packs of pencils. Each pack has 16 pencils in it. How many pencils are there in 5 packs?

Sample solution 2:



Shama spent Rs. 16 to buy a notebook and Rs. 5 to buy a pen. How much did she spend altogether?

While assessing, it is important to focus on the child's facility in relating operations to a given situation, and not on other aspects. In the samples shown, the first child has made minor spelling errors, but demonstrates a clear understanding of the task and can match a situation correctly to the operation given. The second child is not clear about what situation matches the given operation. So, the first child demonstrates a better understanding of the concept that is being assessed here.

ACTIVITY 35

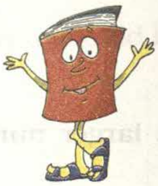
Conceptual area: Relating operations to situations.

Material: Paper and pencil.

Instructions: Give problems like–

(a) 120 school children are going to the zoo. They have to go by vans. Each van can accommodate 30 children. How many vans are required?

(b) 135 school children are going to the zoo. They have to go by vans. Each van can accommodate 30 children. How many vans are required?



Part (b) of the problem provides an excellent opportunity for assessing problem-solving skill. Here after dividing, the child will have to reason that even if only a few children are left over, an extra bus will be required for them. So the child should be able to work out that the answer is 5 vans, and able to provide justification for it.

3.2.2 Fluency in computation (Multiplication and Division).

Multiplication tables tend to occupy a prime place in the learning of multiplication and division in schools. There is little doubt that knowing the tables is useful not only in computation but also later in developing analytical abilities to recognise and create number patterns. But this does not mean that tables should be learnt by rote. The process of learning tables can be challenging as well as fun for the children. You can ask your children to construct their own multiplication tables using number boards.

After mastering 'tables', or rather, the basic multiplication facts, the children have to go on to multiplication procedures for larger numbers. The following ideas are important for children to know in order to understand formal algorithms for 2-digit by 1-digit and larger multiplications.

For multiplication:

- Distributive law
- Multiplication by 10, 100
- Place value concept
- Basic multiplication facts

For division:

- Basic multiplication facts
- Place value concept
- Division by 10 and 100
- Multiplication by 2-digit numbers

While assessing, it is important to check that the child demonstrates an understanding of the multiplication and division they are doing, rather than just having learnt the procedures by rote – without knowing clearly what they really mean. Children also need to be assessed for their ability to use properties (like commutativity and associativity, though knowing names is not necessary for children) to compute more easily.

ACTIVITY 36

Conceptual area: Fluency in computation (Knowledge of basic facts).

Material: Pebbles or counters.

Instructions

(a) Ask the child to use the material to construct multiplication tables of 2, 10, 3, 5 etc.

(b) Once they understand the principle of counting in equal sized groups – that is, counting in 2's, counting in 5's etc., they can be assessed for the knowledge of the basic multiplication facts.

ACTIVITY 37

Conceptual area: Fluency in computation (Understanding and using standard algorithms).

Material: Paper and pencil.

Instructions

Example: 34 = 30 + 4

So, 34 × 7 = (30 × 7) + (4 × 7) = 210 + 28 = 238

Now, find the following products by breaking the larger number into hundreds, tens and ones:

(a) 23 × 8 = (20 × 8) + (3 × 8) = ______
(b) 45 × 6 = (__ × 6) + (5 × 6) = ______
(c) 213 × 7= (200 × 7) + (10 × 7) + (__ × 7) = ______

The child is expected to multiply numbers by a single digit number using distributive law, as shown in this Activity even before they learn formal algorithms for multi-digit multiplication. Although every child may not master this concept fully, it is important for them to have an idea of why we follow a certain sequence of steps while multiplying or dividing. Those with a deeper understanding of the rationale are likely to develop better facility with number operations.

ACTIVITY 38

Conceptual area: Fluency in computation (Understanding the operation).

Material: Paper and pencil.

Instructions

If 4 × 8 is 32, what is 4 × 9?

If 4 × 14 is 56 what is 4 × 15?

If 13 × 241 is 3133, what is 13 × 242?

ACTIVITY 39

Conceptual area: Fluency in computation (Understanding and using standard algorithms).

Material: Paper and pencil.

Instructions: Ask the child to provide you with the missing digits in the following problem.

		2	5	
×		1	3	
		*	5	
	2	5	0	
	*	2	5	

For the children who are not able to handle such tasks independently yet, you can ask questions like the following to guide them:

(Related to the first missing digit in the problem)

- What is 3 x 5?
- Why do we only write 5 in the units place? What is the place value of the '1' in 15? (Note if the child understands the fact that 10 units is equal to 1 ten and hence has to be regrouped and added to the tens place)

You may ask similar questions to check for the understanding of regrouping while adding partial products. Similar questions can be asked for division as well.

You should look out for errors arising due to zeroes, like:

	1	0	5
x		3	4
	4	0	0
3	0	5	0
3	4	5	0

	2	1	3
x	1	0	2
	4	2	6
2	1	3	0
2	5	5	6

ACTIVITY 40

Conceptual area: Fluency in computation (Understanding and using standard algorithms).

Material: Paper and pencil

Instructions: Problems of increasing levels of difficulty like the following can be asked with different level of facility with division:

(a) 72 ÷ 9

(b) 78 ÷ 6

(c) 426 ÷ 3

(d) 709 ÷ 7

(e) 216 ÷ 2

(f) 2008 ÷ 8

Zero alert! In a problem like this involving zero, children may not consider zeroes for division as shown in the box below. They tend to consider only the non-zero numbers for division. They do not realise the importance of zero as a place holder here.

	2	6			
8)	2	0	0	8	
−	1	6			**Step 1**
		4	8		
	−	4	8		**Step 2**
		0	0		

ACTIVITY 41

Conceptual area: Fluency in computations (Understanding and using standard algorithms).

Material: Paper and pencil.

Instructions: Multiply 123 × 645

Possible child error:

	1	2	3
x	6	4	5
	6	1	5
	4	9	2
	7	3	8
1	8	4	5

		1	2	3
	x	6	4	5
		6	1	5
	4	9	2	0
7	3	8	0	0
7	9	3	3	5

In this case, saying that the child has an insufficient understanding of the procedure of carrying out multi-digit multiplication. She has probably not understood that while multiplying 123 by 645, you are actually multiplying 123 by 600, and not 123 by 6. The child may need support in understanding how to use place value in multi-digit multiplication. Some ways in which this can be done are:

- Use exercises like 645 = ( ) × 100 + ( ) × 10 + ( ) ×1 to assess the concept of place value. Teacher gives children different parts of the problem to solve and then asked them to put it together.
- Give problems as in Sample Activity 23.
- Display the problem with an error to the child and ask to find error in it and explain it at the conceptual level. Here children find the procedural problem first and than go on explaining it conceptually.

4.3 Using Problem Solving Strategies

Number operations can be very useful in developing problem solving abilities in children. Hence, an important part of assessment in Number Operations is to engage and assess the child in various problem solving situations. With time, a child should demonstrate improvement in her skill in attempting problems, and also confidence in her own ability to tackle handling problem solving situations.

You could first start by telling the child to solve the problem in her mind. But, if the child expresses her wish to write and solve it, she should be free to do so. Counter and paper pencil should be placed near by. If the child has not used paper pencil to solve the problem, after her answer you may also ask her: 'Can you also solve it using paper and pencil?' and see how she represents the problem. Two sample situations below represent how you may proceed with assessing the child's conceptual understanding.

Teacher : Richa had 5 pencils. Her father gave her few more. Now she has 11 pencils. How many pencils did her father give her?

Child : (*Thinks for sometime*) 6 pencils.

Teacher : How did you do this? Can you show it to me?

Child: Ok! Richa had five pencils (shows 5 fingers on right hand) and then some more got added to them (shows through action of left hand how some pencils got added to the right hand) to make total 11 pencils. So, 5 plus 5 is 10 and if we add another 1, it becomes 11. So, if we add 6 pencils to 5 pencils we get 11 pencils. Hence, Richa's father gave her 6 pencils.

Discussion: This answer gives you evidence that the child has the notion of the unknown quantity and is hence able to model the 'join' situation mentally. Thus, you may now be interested to know if the child can identify this structure in symbolic representation situations as well.

Teacher: Shows the following representations to the child and asks to identify which one represents the above problem.

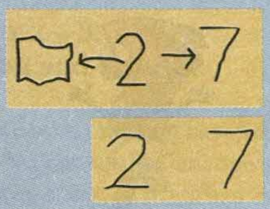
a) 5 + 11 = __ ?

b) 5 + __ = 11 ?

Child: (b) is the correct answer.

Discussion: The teacher has now confirmed that the child can understand the structure of the problem when it is given to the child without the context only in terms of mathematical symbols. She is able to identify that (b) represents the structure of the word problem.

In order to confirm the understanding of her the teacher asks the child to formulate another problem.



□←2→7

2 7

Teacher: Can you formulate more questions of the form 3 + __ = 9?

Child: I had 3 mangoes and my mother gave me some more. Now I have 9 mangoes. How many mangoes did my mother give me?

Discussion: The above responses show that the child has understood the structure of the 'add to' problems and is thus able to construct new problems and also identify the structure of the problem when given in form of mathematical symbols.

Situation 2:

Teacher: Harpreet had some marbles. Her friend Rekha gave her 2 more. Now she had 7 marbles. How many marbles did Harpreet have to start with?

Child : *(shows confusion, uses the pen and paper to write)*

Teacher: I am sure you will be able to solve this problem, try it once.

Child: (*fumbles*) I don't know.

Discussion: The teacher understands that the child is still unable to mentally model the situation and thus scaffolds the child's thinking by representing the problem for him.

Teacher : Ok! I'll give a small hint. I have some marbles hidden in my hand and I add two more to them. (*The teacher takes a set of marbles and keeps 5 of them in her hand and asks the child to give her 2 more marbles*). Now I have 7 marbles altogether in my hand. How many marbles did I have to start with? Do you want to show the problem on paper?

Child: Represents the problem as follows-

Discussion: In the above situation the teacher finds out that the child is unable to model the situation mentally on his own so the teacher scaffolds the child by modelling this problem for the child through using concrete objects. The child is thus able to model the situation on paper. In case the child had been unable to model the situation even after help from the teacher, the teacher may decide to ask the child simpler addition type problems where the result is unknown.

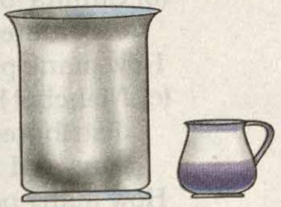
You may tone down the level of a task by modelling the situation using pictures or in concrete mode according to the understanding exhibited by the child.

ACTIVITY 42

Conceptual area: Using problem solving strategies.

Material: Water, a large container, a cup.

Instructions: Call out a group of children, say 5. Ask each child to pour 2 cups of water into the container one by one.



Then ask them how many cups of water the container now has. When children give their answer, ask them how they found out.

Repeat the same problem and this time ask children to pour three cups each, five cups each, etc.

Note the techniques children evolve to represent the problem to themselves and find the answer.

Observe if they are noting any patterns and making any generalisations. Also note when they begin to 'find the answer' even before or without actually pouring water into the container.

The following example shows how you may assess the child's ability to model a multiplication situation.

Teacher: There are 5 children and each one pours 3 cups of water in the container. Then how many cups of water are there in the container?

Child: (*replies*) 15.

Teacher: How did you find that out?

Child: By thinking about it.

Teacher: No, but I think the answer is 17, don't you think so?

Child: No, how can it be? See, if I put 3 cups, then you put another 3, that is 6 cups, if now Shyam puts another 3, that is - 7,8,9 cups, then another 3 cups put by Hari, that means 9 cups and another 3 by Soma. In total there are 15 cups.

Discussion

This situation gives you evidence that the child has modelled the situation of multiplication correctly and you may now ask the child to create a new problem of a similar structure.

In case the child provides an answer (for e.g. 3 cups + 5 children = 8 cups), she has been unable to model the situation and you may scaffold her thinking by helping her draw the children and the cups of water poured by each on paper or by carrying out the activity physically.

ACTIVITY 43

Conceptual area: Using problem solving strategies.

Material: Paper and pencil.

Instructions: Give the children 'story' problems based on more than one operation. For instance:

(a) Aaloka had 27 books. One day, 8 of them got wet in the rain and got spoilt. The next day, her mother bought her 10 new books. How many books does she have now?

(b) Sumita bought 8 packets of pencils containing 6 pencils each. She distributed the pencils equally to the 24 children in her class. How many pencils did each child get?

(c) Nishchal bought some packets of colour pens containing 6 pens each. He distributed the pens equally to 20 childs in his class. Each child got 2 pens and 1 pen was left over. How many packets of pens did Nishchal buy?

ACTIVITY 44

Conceptual area: Using problem solving strategies.

Material: Paper and pencil.

Instructions: Divide the class into groups and ask each group to make problems that involve 2 or 3 operations. Initially the number sentences may be given.

(a) Make a 'story' that fits the number sentence 5 x 2 + 3.

(b) Make a number story that involves both addition and division.

The problems created by groups can be assessed based on whether they mathematically fit the number sentence given, and also the variety and appropriateness of the situation used.

Later, the groups can exchange these problems and solve them. So each group can be assessed on creation of the problems and also, solving problems formed by the others.

Note that in every situation where the child has to match situation with the correct operation, her problem solving skills come into play. Hence all the tasks in the sections discussing 'Relating operations to situation' can be used to assess problem solving skills also. Even some tasks in the computations section – where the child is asked to develop or analyse a new algorithm for an operation based on what he already knows, can double up as tasks to assess problem solving skills.

4.4 Indicators

The indicators for assessing conceptual understanding and reasoning skills related to Number Operations for Classes I-V are presented in the table below. The teachers can develop their own indicators based on the syllabus.

	Conceptual area	Sample Activities	Proficiency Level		
			Basic	Intermediate	Advanced
Addition and Subtraction (4.1)	Relating operation to situations	15 to 21	Demonstrates an initial understanding of addition and subtraction situations. Occasionally able to represent simple addition or subtraction situations correctly using mathematical operations and symbols*(Sample Activities: 15-20)*. May require support to match or create 'stories'/ situations for a given addition or subtraction fact, like 5 + 4 = 9 *(Sample Activity: 21)*.	Demonstrates an adequate understanding of addition and subtraction situations and the relation between the operations. Mostly able to represent simple addition or subtraction situations correctly using mathematical operations and symbol. May have trouble with the tougher cases – may get confused with keywords etc. (for example, may do 7 + 2 in the problem "Ali has 7 kites. He has 2 kites more than Rani. How many kites does Rani have?"*(Sample Activities: 15-20)*. Mostly able to match or create 'stories'/situations for a given addition or subtraction fact, like 5 + 4 = 9 *(Sample Activity: 21)*.	Demonstrates a good understanding of additon and subtraction situations and the relation between the operations.Consistently able to represent addition or subtraction situations correctly using mathematical operations and symbols, even in the tougher cases, without getting confused with keywords etc. *(Sample Activities: 15-20)* Consistently able to match or create 'stories'/situations for a given addition or subtraction fact. Creates stories of different types. For instance, compares problem with smaller value unknown for a given addition fact like 5 + 4 = 9. (for example, "Sita read 5 books in the holidays. She read 4 books less than Rita. How many books did Rita read?" *(Sample Activity: 21)*.
	Fluency in computation	22 to 27	Demonstrates initial knowledge of basic single digit addition-subtraction facts – may need to use concrete objects to add or subtract two single-digit numbers *(Sample Activities: 22-24)*. Requires support to use standard algorithms (procedures) to add or subtract numbers *(Sample Activities: 25-27)*.	Demonstrates adequate knowledge of basic single digit addition-subtraction facts – may occasionally need to use concrete objects to add or subtract two single-digit numbers *(Sample Activities: 22-24)*. Mostly able to use standard algorithms (procedures) to add or subtract numbers correctly. May make errors in certain 'difficult' types, like subtracting from a number with zeroes, for example: 200 - 35 *(Sample Activities: 25-27)*.	Demonstrates mastery over basic single digit addition-subtraction facts – consistently adds or subtracts any two single-digit numbers without the help of concrete objects *(Sample Activities: 22-24)*. Consistently able to use standard algorithms (procedures) to add or subtract numbers correctly*(Sample Activities: 25-27)*. Able to generalise and extend known procedures in the case of larger numbers. For instance, after having learnt addition with regrouping for 2-digit numbers, is able to extend it for 3-digit numbers*(Sample Activities: 25-27)*. Consistently able to use different strategies like properties and shortcuts to compute mentally, in any problem and in real life situations *(Sample Activity: 27)*.

	Conceptual area	Sample Activities	Proficiency Level		
			Basic	Intermediate	Advanced
Multiplication and Division (4.2)	Relating operation to situations	28 to 35	Demonstrates an initial understanding of multiplication and division situations *(Sample Activities: 28-30).* Occasionally able to represent simple multiplication or division situations correctly using mathematical operations and symbols *(Sample Activities: 31-33, 35).* May require support to match or create 'stories'/ situations for a given multiplication or division fact, like 5 × 3 = 15 *(Sample Activity: 34).*	Demonstrates an adequate understanding of multiplication and division situations and the relation between the operations *(Sample Activities: 28-30).* Mostly able to represent simple multiplication or division situations correctly using mathematical operations and symbol. May have trouble with the tougher cases – may get confused with keywords etc. *(Sample Activities: 31-33, 35).* Mostly able to match or create 'stories'/situations for a given multiplication or division fact, like 5 × 3 = 15 *(Sample Activity: 34).*	Demonstrates a good understanding of multiplication and division situations and the relation between the operations. *(Sample Activities: 28-30).* Consistently able to represent any multiplication or division situation correctly using mathematical operations and symbols, even in the tougher cases, without getting confused with keywords etc. *(Sample Activities: 31-33, 35).* Consistently able to match or create 'stories'/situations for a given multiplication or division fact. Creates stories of different types. (for example, make a problem like "How many packs of 2 can you make from 16 sweets?" when asked to create a story for the situation 16 ÷ 2 = 8) *(Sample Activity: 34).*
	Fluency in computation	36 to 41	Demonstrates initial knowledge of multiplication tables *(Sample Activities: 36, 38)* Occasionally able to use standard algorithms (procedures) to multiply or divide numbers correctly *(Sample Activities: 37, 39-41).*	Demonstrates adequate knowledge of multiplication tables (of numbers up to 10) but may occasionally make mistakes *(Sample Activities: 36, 38)* Mostly able to use standard algorithms (procedures) to multiply or divide numbers correctly *(Sample Activities: 37, 39-41).*	Demonstrates complete knowledge of the multiplication tables (of numbers up to 10) and mastery over all basic multiplication facts. May use own strategies to develop tables of larger numbers etc. *(Sample Activities: 36, 38)* Understands the rationale of standard algorithms (procedures) and is consistently able to use them to multiply or divide numbers correctly *(Sample Activities: 37, 39-41).* Able to generalise and extend known procedures in the case of larger numbers. Or, may develop new alternate algorithms for larger numbers Consistently able to use different strategies like properties and shortcuts to compute mentally.
Using problem solving strategies (4.3)	Relating operation to situations	42 to 44	Sometimes able to solve problems independently – may mostly require support to understand the problem or formulate strategy. Requires support to continue with the problem solving task.	Sometimes able to solve problems effectively – may require support to understand the problem or formulate strategy when new problem types are given. Moving towards appropriate attitude and behaviour during problem solving tasks.	Consistently able to solve problems effectively, by using a range of procedures and strategies. Consistently demonstrates appropriate attitude and behaviour during problem solving tasks – formulating strategy carefully, reviews strategy if it takes too long to solve, not giving up too easily, providing proper justification for the answer, checking the answer etc.

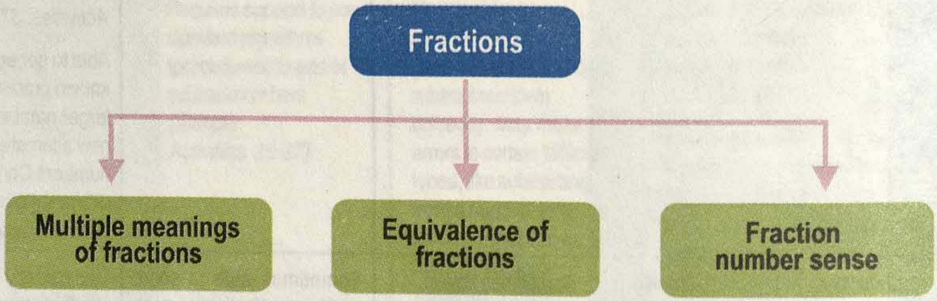
5

Fractions

Learning about fractions and decimal numbers is important for a primary school child because these expand her understanding of numbers – to include quantities that are not whole. In the primary grades, children need to develop an understanding of fractions as a quantity, and as division or ratio. Fraction operations need not be stressed at this level, rather, children should develop 'fraction number sense' – the ability to handle fractional quantities with ease.

Learning about fractions in the primary grades means the development of concepts and skills in three broad categories. The three categories are:

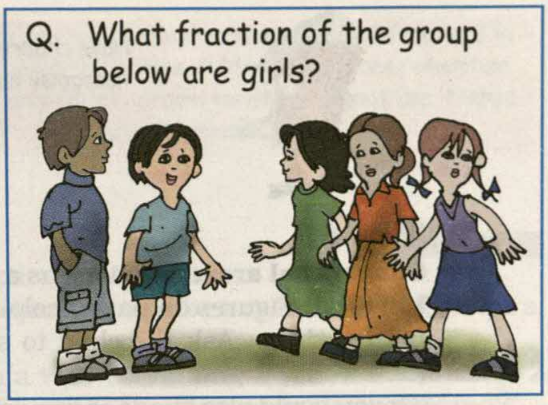
- Understanding the multiple meaning of fractions: This would involve a basic understanding of the complex nature of fractions – with a perspective on its many different meanings.
- Understanding and using equivalence of fractions: A lot of the advanced concepts in fractions depend on the idea of equivalence – for instance, comparison addition and subtraction. Even the idea of decimals and percentages are directly based on the idea of equivalence – equivalence with specific denominators (Powers of 10 in the case of decimals, and 100 in the case of percentages).
- Developing Fraction 'Number Sense': Children may go through the motions of learning fractions and decimals without developing any idea of what they really mean. So, an important goal of learning about fractions is to be able to make sense of fractional quantities. This would include being able to estimate a whole number equivalent of a fractional number, and being able to think and reason in terms of unit fractions.



5.1 Understanding multiple meanings of fractions

Usually, children are introduced to fractions with 'part of a whole' examples. Even in this, only one type of problem is usually taken up: "Shade ½ of this figure' being one of the most common. While understanding fractions as a part of a whole, some of the key understandings are – the size of the constituent parts must be of equal size, the concept of a unit fraction etc.

However, for a complete understanding of fractions for a primary school child, she must understand other meanings of fractions. This means, not limiting the child to the 'part of a whole' meaning of a fraction as is normally done, but an introduction to other ideas – like fractions as part of a collection, fractions as a division operation, or for comparison.


Q. What fraction of the group below are girls?

Sometimes, too much stress is put on learning of terms (like numerator, denominator, like and unlike fractions etc.) at the cost of developing these key concepts. For instance, a child might be able to 'define' the terms numerator and denominator accurately, but when asked what part of a group with 3 boys and 2 girls are – girls, say that it is 2/3. This displays a faulty understanding of what the denominator really means.

The items below show how to focus on key concepts while assessing the child's understanding of fractions and fraction representation.

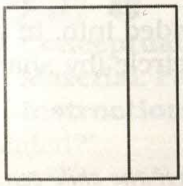
ACTIVITY 45

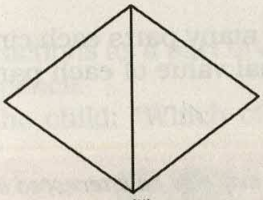
Conceptual Area: Fractions as part of a whole.

Material: Figures on Paper.

Instructions

(a) Which of the following figures are divided into two equal parts?


(i)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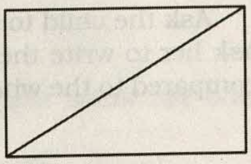
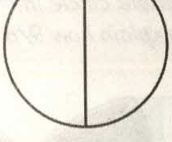
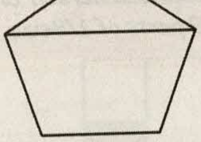

(iii)


(iv)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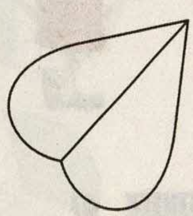
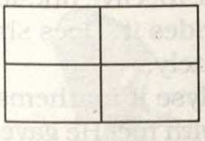
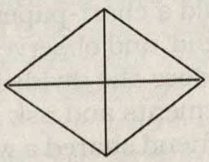
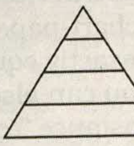

(vi)

(b) Which of the following figures are divided into 4 equal parts?


(i)


(ii)


(iii)

Have children understood what 'equal parts' mean? Can they recognise equal parts even when they are flipped over as in Item 1(a) part (iii)?

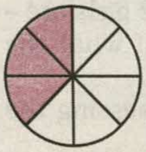
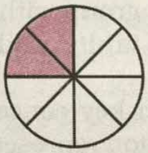
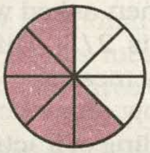
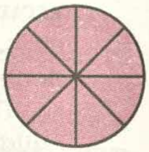
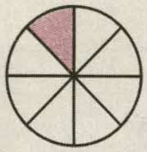
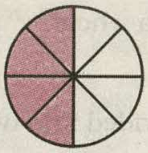
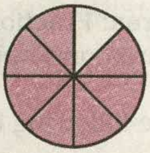
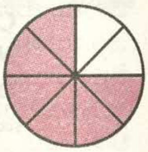
ACTIVITY 46

Conceptual area: Fractions as a part of a whole.

Material: Figures on paper, colours.

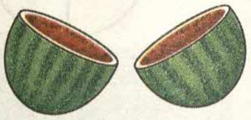
Instructions: Ask the child to shade the following fractions in the given circles.

(Or you could also shade as required and ask the child to write an appropriate fraction.)

Ask the child to say how many parts each circle is divided into. In addition ask her to write the fractional value of each part of the circle (by shading) as compared to the whole.

You may also be interested in assessing if the child can shade 9/8. This will help you understand if the child is able to understand the idea of having two (or more) wholes for fractions greater than 1. You may, ask the child to shade 9/8 and note if she herself draws another whole circle in order to shade 9 parts of 1/8 size. Ask them to explain how $9/8 = 1\frac{1}{8}$.



ACTIVITY 47

Conceptual Area: Fractions as part of a whole.

Material: Any material present in the classroom, which can be used for dividing or sharing – biscuit, chart-paper etc.

Instructions: Give a child a chart-paper and ask her to 'Give one-fourth of this chart-paper to your friend' and observe how she divides it. Does she make four exactly equal parts, or does she divide it approximately.

You can also make statements and ask them to analyse it mathematically. For instance 'Yesterday my friend shared a watermelon with me. He gave me the bigger half.' Ask them whether this statement is mathematically correct, and if not, what the error is, and give reasons.

It is important to check for an understanding of fractions in context sometimes - otherwise children sometimes remember certain things only for classroom tasks and do not use it when actually needed!

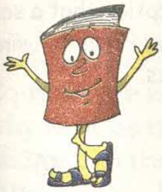
ACTIVITY 48

Conceptual Area: Fraction as a part of a collection.

Material: Collection of different objects – could be a vase with some flowers, for instance.

Instructions: Show the children a vase containing 5 yellow flowers and 2 red flowers. Ask the children, 'What fraction of the flowers in the vase is red?".

The child is expected to say: 'Two-seventh.' Some children may say: 'Two-fifth'. Such children lack an understanding of fractions as a part of a collection, in not taking into account all the objects in the set. Later, a collection of objects with 3 or more types can be used.



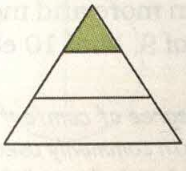
Fraction as a part of a collection is an important understanding for children, since this is probably used more often in real life as we say, for example, "Almost one third of the class is absent today". Misconceptions like the one in the item above often happen if we focus only on the 'parts of a whole' model.

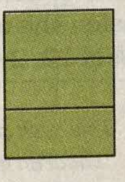
ACTIVITY 49

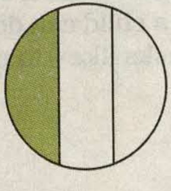
Conceptual Area: Fractions as a part of a whole.

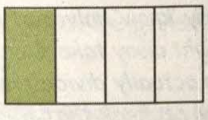
Material: Paper and pencil.

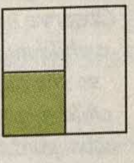
Instructions: Ask the child: "Which of the figures below are one third shaded?"

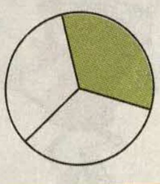


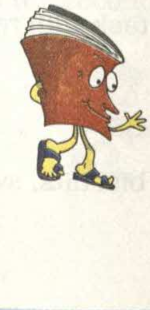












This item addresses the understanding that constituent parts need to be equal. It also addresses several misconceptions - so for each figure, a child should be asked to justify why she says the shaded part is one-third or not. Some children may show a flawed understanding of 'denominator' as total number of constituent parts, by counting the unshaded parts only, and choosing figure D as being 1/3 shaded.

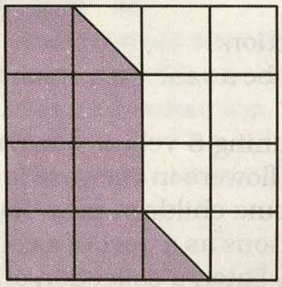
ACTIVITY 50

Conceptual Area: Fractions as a part of a whole.

Material: Squared sheets.

Instructions: Ask the children to draw many 4 by 4 squares, and shade half of each, in as many different ways they can think of.

Sample Child Solution:

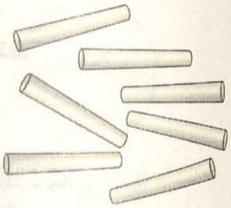


The activity can be extended to have children draw different shapes on the squared sheet, and shade exactly half of each. A child should be able to justify why the part she has shaded is exactly half.

The activity can also be extended for other common fractions – such as ¼ and ¾ also.

The first task also addresses the misconception that a square can be divided into half in a few set ways - mostly using vertical or horizontal lines.

ACTIVITY 51



Conceptual area: Fraction as part of a collection.

Material: A collection of objects – for instance toffees, chalks, crayons etc.

Instructions

(a) Display a small set of objects from the collection – say 8 pieces of toffees. Ask a child to pick out half of them. If the child is successful, she can be asked to pick out a quarter of them.

(b) If a child can do these tasks, she can be given more and more challenging tasks like – to pick out ¾ of 12, 2/3 of 9, ½ of 9, ¼.of 10 etc.

This task checks the degree of comfort that individual children might have with commonly used fractions by asking them to pick out $\frac{1}{2}$, $\frac{1}{4}$ etc. of a collection. Observe how the child attempts to do it. For instance, a child might already 'know' halves of some numbers - so she might straight away take 4 toffees, Another child might have to actually divide physically into two equal parts, and then recheck that the parts are equal. Observe the procedures used for $\frac{1}{4}$ and $\frac{3}{4}$.

This task could be used as a colouring task instead of a task with real objects by drawing pictures of sweets etc. on a paper or a slate.

ACTIVITY 52

Conceptual Area: Fraction as division.

Material: Any material available for dividing – chalks, biscuits, sweets etc.

Instructions

(a) Give the child 1 biscuit, and ask her to divide it equally between 2 children. Then ask her: "What part of the biscuit did each child get?"

(b) Give the child 1 biscuit, and ask her to divide it equally between 4 children. Then ask her: "What part of the biscuit did each child get?"

(c) Give the child 3 biscuits, and ask her to divide it equally between 4 people. Then ask her: "How much is each person's share?"

In each case ask them how they found the answer. They should demonstrate the understanding that 1 ÷ 2 = 1/2, 1 ÷ 4 = 1/4 etc. If some child can not figure this out, ask him simpler questions like "If 10 *rotis* have to be divided amongst 5 people, then how will you find out each person's share?" Children would know that they have used 'division operation' (10 ÷ 5) in order find equal share. This can then be related to the division problems asked.

Later, more difficult sharing problems: could be asked.

5.2 Equivalence of fractions

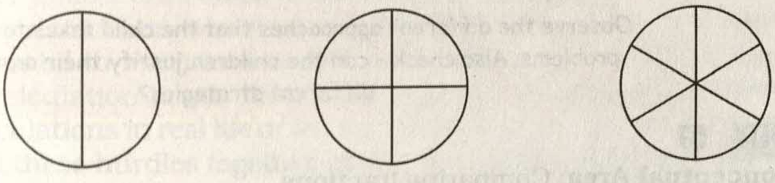
Equivalence is one of the key ideas in fractions. This helps the child to understand why various numbers that 'look' very different – like 1/2, 3/6 or 50/100 actually amount to the same thing. The idea of equivalence is also useful in developing other skills – like comparing fractions, and later on, addition and subtraction of fractions.

ACTIVITY 53

Conceptual area: Equivalence of fractions.

Material: Paper and pencil, colours.

Instructions: Show 3 circles of equal size to the child, as shown below. Ask him to colour 1/2 of each.



After she has shaded, ask the child what conclusion she can make from this. They should be able to conclude that 1/2 = 2/4 = 3/6.

Later, they can be given square grids (like the one used in Activity 50).

ACTIVITY 54

Conceptual Area: Equivalence of fractions.

Material: Two equal pieces of chart paper.

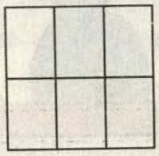
Instructions: Tell the children "I will give Rani 2/3 of the first paper and Ali 4/6 of the second. Whose paper will be bigger? Why?"

Encourage the children to try and predict with reasons, without physically dividing the papers. Observe how many can reason correctly that way. Later they can be allowed to actually divide the paper and check.

ACTIVITY 55

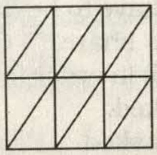
Conceptual area: Equivalence of fractions.
Material: Figures on paper, colours.
Instructions: Ask the child to shade 2/3 of the square given below.

(A)



If a child finds figures like this (A) easy, slightly more challenging ones (like B) can be given.

(B)



Using the idea of equivalence is implicit in the problem without stating it. If the child is successful in such tasks, it means she is able to use the idea of equivalence well!

ACTIVITY 56

Conceptual Area: Equivalence of fractions.
Material: Paper and pencil.
Instructions: Name some fractions equivalent to 8/12.

Related questions like "What is the lowest form of the fraction 8/12?" etc can also be asked.

If the child does the tasks successfully, you can probe more to check for deeper understanding - "Can you draw or use a paper to show me how they are equivalent?"

How is the child trying to find the answer? Is she multiplying 8/12 with 1, 2,... or trying few fractions such as 2/3 or any other fraction multiplied with 1, 2, 3...

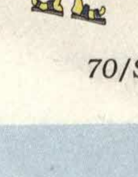
Observe the different approaches that the child takes to solve these problems. Also check - can the children justify their answers using different strategies?

ACTIVITY 57

Conceptual Area: Comparing fractions
Material: Oral task
Instructions

Ask the child: "Which is more: 1/9 or 1/10? Why? Explain in two different ways."

Later, the task might be extended – the children can be asked to reason whether 7/9 is greater or 7/10., etc.



This task is meant to give an opportunity for children to apply their understanding of equivalence. Asking for alternate ways of explaining also might be a good idea. Apart from the traditional 'writing as equivalent fractions' method, diverse child solutions should be encouraged and accounted for in the assessment. For instance, in case of the fractions 1/9 and 1/10, if a child says, 'If the number of parts in which the whole thing is divided is more, the size of each part will decrease. So 1/10 is less than 1/9', she shows a good conceptual grasp of fractions!

ACTIVITY 58

Conceptual Area: Comparing fractions.

Material: Paper and pencil.

Instructions: For each of the following, circle the larger of the two. In case they are equal write '=' between them. Give reasons for your answer.

(a) $\frac{1}{6}$ $\frac{1}{8}$

(b) $\frac{1}{7}$ $\frac{2}{7}$

(c) 1 $\frac{4}{4}$

(d) $\frac{3}{6}$ $\frac{1}{1}$

(e) $\frac{4}{5}$ 1

You may ask the child to show how she has arrived at the solution of comparing the two given fractional numbers. Ask her, "Can you draw on or use a paper to show me how you compared these fractional numbers?"

This can be used to assess the ability of the child to compare two fractions.

5.3 Fraction number sense

Many children's difficulties in mathematics start with the introduction of fractions. The reasons for this may be manifold. Many ideas related to whole numbers may get applied wrongly in the case of fractions – for instance, a child reasoning that 1/2 is less than 1/3 because 2 is less than 3. Also, compared to whole numbers, children spend very little time on the fundamental concepts of fractions, often moving quickly onto learning for operations and their properties. A third reason for child's difficulties is the tendency to computations and other work with 'bigger' fractions, which have no bearing on the child's life. What use is it to a young child to learn how to add 3/13 and 2/15? How often have we done such a calculation in our life? In fact hardly any of us would have ever done such calculations in real life or felt the need for them – except in the mathematics class! All these hurdles together hinder the child's understanding of fractional quantities.

How many times you have faced a situation where you have had to compute something like 3/13 + 2/15? Most likely, never! Why burden young children with calculations involving fraction before they are ready, then?

Keeping this in mind, it is desirable to keep the focus of the assessment of fractions on the fundamental concepts using commonly used fractions like 1/4, 1/2, 3/4 etc. It is recommended that at this level, conceptual work with commonly used fractions like 1/2, 1/4, 3/4 etc. should be focused on, leaving formal algorithms for fraction operations and other mechanical work with 'large looking' fractions well alone.

So, the assessment should be geared more towards checking the child's level of comfort with handling commonly used fractions like ½, ¼, ¾ etc, and informal reasoning with fractional quantities – and not rigid algorithms.

ACTIVITY 59

Conceptual area: Concept of whole.

Material: Oral task.

Instructions: Ask the children: "Latha had half a *paratha* and Kartik also had half a *paratha*. But Latha said she had more *paratha* than Kartik. Can Latha be right? Give reasons and draw pictures if necessary to support your answer."

The size of the two halves depends on the whole they are a part of. This task checks for the understanding that one cannot compare real quantities only by looking at the fraction that is assigned to the share, but also has to consider the size of the two wholes. Children who have understood this might reason that 'Yes, it is possible, because Latha's *paratha* might have been bigger than Kartik's. Think of some other tasks like this to check such understanding!

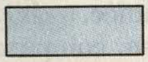
ACTIVITY 60

Conceptual area: Concept of whole.

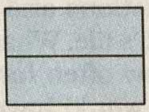
Material: Paper and pencil, or, figures on dot paper.

Instructions

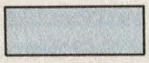
(a) This is half of some whole.



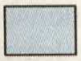
Then, which of the following is the whole?



(A)

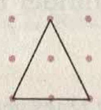


(B)

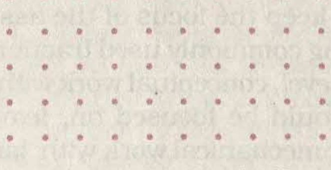


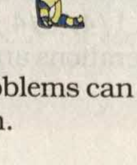
(C)

(b) Later, more advanced tasks can be given on dot paper, like this one:
If this is ¼ of some whole,



then what is the whole?
Draw it on the grid below:





How well do the children understand the meaning of a whole? They probably know how to find a fraction when the whole is given - but can they construct the whole when a fraction of the whole is given?

(c) Later, even more challenging problems can be given – for example, finding the whole when 2/3 of it is given.

ACTIVITY 61

Conceptual area: Working with commonly used fractions - 1/2, 1/4, 3/4 etc. Relations like 1/2 + 1/2= 1

1/4+ 1/4 = 1/2

1 – 3/4 = 1/4

1/4 + 1/2 = 3/4 to be written in large child friendly script in interesting arrangement

Material: Oral task.

Instructions: Ask questions that help the child examine and understand the relationship between 1/2 and 1, 1/4 and 1, 1/2 and 1/4, 1/4 and 3/4 etc using real life situations. For instance:

I bought 1/2 kg of sugar and used 1/4 kg to make *sheera.* How many kg of sugar is left with me?

or

My uncle gave me a melon yesterday. I ate 1/2 of it and gave 1/4 to my friend. What part of the melon is now left?

or

Amina had 2 1/2 kg of rice. Some guests came to visit her. She cooked 3/4 kg rice for them in the morning, and 1/2 kg in the evening. How much rice is left with her now?

These problems can help you assess whether children are able to handle commonly used fractional quantities with ease. Initially, if they may need to work with actual material to figure out that 1/2+ 1/4 = 3/4 etc. But later, they should be able to reason about these quantities in abstract also.

ACTIVITY 62

Conceptual area: Estimating fractional quantities.

Material: Paper and pencil.

Instructions: For each of the following fractions, estimate whether its value is nearly equal to 1/2, or 1, or neither.

6/7 55/54 6/13 19/20 41/80

Check what strategies the children use. Are they figuring out that 6/13 is actually very close to 1/2? If yes, then how? Can they recognise that 55/54 is actually a number whose value is very close to 1? For each number, ask them to tell you how they figured out whether it was close to 1/2 or 1 or neither.

ACTIVITY 63

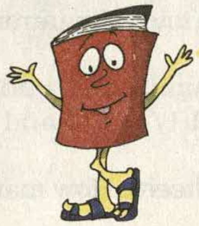
Conceptual area: Estimating fractional quantities.

Material: Paper and pencil.

Instructions: Ask the child to write each of the following fractions in the boxes given below.

5/6 6/11 17/17 111/112 4/3 34/37 3 1/2 100/100

Less than 1	Equal to 1	Greater than 1



Have you noticed the various misconceptions that children have related to fractional quantities? For instance, they may not recognise that 17/17 is equal to 1. Or, they may not realise that 111/112 is actually a quantity less than 1! They may think this way because they think the numbers 17, 111, or 112 determine the value of the fraction rather than 111/112 as a whole!

Apart from checking for understanding of fractional numbers and estimating their value, this item helps in checking for various misconceptions (listed above) related to fractional quantities.

5.4 Indicators

The indicators for assessing achievement in fraction concepts for Classes I-V are presented at the end of the chapter table. The teachers can develop their own indicators based on the syllabus.

	Conceptual area	Sample Activities	Proficiency Level		
			Basic	Intermediate	Advanced
Understanding multiple meaning of fractions (5.1)	Fractions as part of a whole	45-47, 49, 50	Occasionally able to do simple fraction representation tasks (like shading 1/3 of a circle) correctly. May make errors like not paying attention to the fact that the parts should be equal etc.*(Sample Activities: 45-47).*	Mostly able to do simple fraction representation tasks correctly*(Sample Activities: 45-47).*	Consistently able to do fraction representation tasks correctly – including complex ones like shading 4/3 *(Sample Activities: 45-47).* Able to handle more complex tasks based on 'Part of a whole' concept. (*(Sample Activities: 49-50)*
	Fractions as part of a collection	48, 51	Occasionally able to use fractions to represent the relation between some part of a group to the whole group *(Sample Activity: 48).*	Mostly able to represent the relation between some part of a group to the whole group using fractions*(Sample Activity: 48).* Mostly able to pick out half or quarter of a given collection of objects, but may get stuck in complex cases like picking out 2/3 of 9 pebbles etc.*(Sample Activity: 51).*	Consistently able to represent the relation between some part of a group to the whole group using fractions *(Sample Activity: 48).* Consistently able to pick out a given fraction of a collection – including complex ones like picking out 2/3 of 9 pebbles etc. *(Sample Activity: 51).* Applies understanding of fractions as a part of a collection in life situations – for example, she might on her own observe, and comment that "1/4 of the children in our class are girls" etc.
	Fractions as division	52	Occasionally able to do simple equal sharing tasks successfully *(Sample Activity: 52).*	Mostly able to do simple equal sharing tasks successfully, occasionally does more complex tasks like dividing 3 biscuits among 2 children *(Sample Activity: 52).*	Consistently able to do equal sharing tasks successfully – including complex ones like dividing 3 biscuits among 4 people etc.*(Sample Activity: 52).* Applies understanding of fractions in division situations in real life on his own, without being asked by the teacher.
Equivalence of fractions (5.2)	Understanding equivalence of fractions	53-56	Occasionally able to relate or recognise equivalent forms of a fraction, but the understanding might be mechanical. She may not be able to justify why 2/4 = ½, for example *(Sample Activities: 53-56).*	Mostly able to relate or recognise equivalent forms of a fraction, and provide justification for it. But, she may not have confidence in applying the idea of equivalence in other contexts *(Sample Activities: 53-56).*	Consistently able to relate or recognise equivalent forms of a fraction, and provide justification for it in more than one way. Also, applies the idea of equivalence in tasks involving fractions – for example, in comparison of fractions *(Sample Activities: 53-56).* Uses the idea of equivalence to solve real life problems.

	Conceptual area	Sample Activities	Proficiency Level		
			Basic	Intermediate	Advanced
Fraction number sense (5.3)	Comparing fractions Items	57-58	Occasionally able to compare fractions – when the comparisons are simple and intuitive, for instance 1/5 and 3/5 *(Sample Activities: 57-58).*	Mostly able to compare fractions – when the comparisons are simple and intuitive. Occasionally able to use the idea of equivalence or other strategies in the case of non-intuitive comparisons like comparing 1/4 and 1/6 *(Sample Activities: 57-58).*	Consistently able to compare fractions - even when the comparisons are non-intuitive *(Sample Activities: 57-58).* Uses a range of different strategies while comparing fractions in classroom tasks or in real life situations – equivalence, unit fractions, diagrams etc.
	Concept of a whole	59-60	Demonstrates an initial understanding of the concept of a whole – as opposed to a 'part'. But may have difficulty visualising what the whole could be when only a part is given, etc. *(Sample Activity: 60).*	Demonstrates an intermediate understanding of the concept of a whole. May be able to visualise what the whole could be when only a part is given in simple cases – for example, when the given part is half of the whole *(Sample Activity: 60).*	Demonstrates an advanced understanding of the concept of a whole. Consistently able to visualise what the whole could be when only a part is given – even in complex cases for example, when 2/3 is given *(Sample Activity: 60).* Able to do advanced reasoning related to concept of whole in classroom tasks, and real life problems(Sample Activity: 59).
	Working with ½ ¼, ¾	61	Occasionally demonstrates understanding of relationship between ½, ¼, ¾, and 1. May know the easier relationships (½ and 1) but have difficulty in the more difficult ones (½ and ¾ etc.) *(Sample Activity: 61).*	Mostly demonstrates understanding of relationship between ½, ¼, ¾, and 1. *(Sample Activity: 61).* Might occasionally use these relationships in problems or real life situations.	Consistently demonstrates understanding of relationship between ½, ¼, ¾, and 1. *(Sample Activity: 61)* Consistently uses these relationships in problems or real life situations.
	Estimating fractional quantities	62-63	Occasionally able to estimate values of common fractional quantities, for instance saying that ¼ and ½ are less than 1. But may have misconceptions like – thinking that the value of 10/11 is close to 10 etc. *(Sample Activities: 62-63).*	Mostly able to estimate values of fractional quantities - showing a fundamental understanding of whether a fraction is less than, equal to, or more than 1, etc. *(Sample Activities: 62-63).*	Consistently able to estimate values of fractional quantities, and demonstrates complete clarity on which whole number the fraction is nearly equal to *(Sample Activities: 62-63)* Confidently uses estimations and reasoning about fractional quantities in real life situations.

6

Shapes and Space

Spatial reasoning skills and a working knowledge of geometry are necessary to understand, interpret and appreciate the world around. A good understanding of shapes could help an individual deal with shapes and space in various ways– use vocabulary related to shapes and space confidently in real-life situations, and use the knowledge of shapes for creative or artistic purposes, among other things.

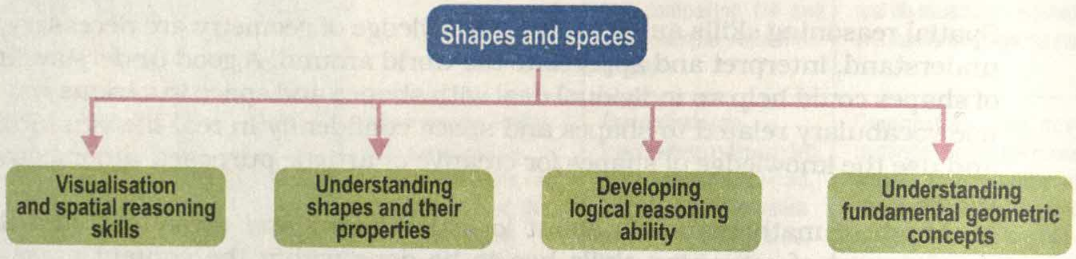
Much of mathematics is about logical thinking and reasoning, and the development of reasoning skills has to be done within the content areas of mathematics. Among the various content areas of mathematics, the one which lends itself most naturally to tasks requiring reasoning and proof, is geometry. Therefore, in addition to creating an understanding about shapes and their properties, one of the main purposes of school geometry is to develop the reasoning abilities of a child. It is critical to keep this in mind while planning any assessment in the content area of Shapes and Space.

Teaching and learning geometry at the primary level aims to develop in a child:

- **Understanding of shapes and their properties:** Geometry begins with identifying and describing shapes. The primary classes are ideal to help children refine and extend the rough, intuitive understanding of shapes that they bring with them as pre-schoolers. It is important to note here, that 'understanding' shapes and space goes much beyond just recognition of simple basic shapes likes rectangles, circles, triangles, cubes etc. The child should develop and use vocabulary related to shapes and space, analyse classes of shapes and their properties etc. in real life contexts.
- **Visualisation and spatial reasoning:** Although knowledge about shapes and their properties is an important aspect of a child's understanding of space, there is another aspect that is equally important. And that is - reasoning about shapes and space. School textbooks tend to focus more on the knowledge aspect, but it is important for you, as a teacher, to see that children gradually become skilled in reasoning with shapes. They should be able to do the following: mentally picturing the result of enlarging, rotating, flipping of shapes, or predicting what a shape would look like from different perspectives.
- **Understanding fundamental geometric concepts**: A primary school child needs to start understanding basic geometric concepts and terms such as the idea of a straight line, parallel lines and angles. This lays the foundation for solving geometrical problems in higher classes.
- **Developing logical reasoning ability:** Developing logical thinking skills is an integral part of geometry and needs to be a part of every geometry classroom, even in classes 1 and 2. This has to take place through tasks

that involve making predictions, drawing conclusions and justifying the same using appropriate vocabulary. For very young children, the tasks can be situated in the ordinary shape sorting and identifying tasks they do.

It is useful to note here that there is a lot of overlap between these four areas. For instance, understanding the concept of angles, parallel lines etc enable a child to have a better understanding of shapes. Logical reasoning is related to all the other areas listed, and indeed to all areas of Mathematics.


Shapes and spaces
Visualisation and spatial reasoning skills
Understanding shapes and their properties
Developing logical reasoning ability
Understanding fundamental geometric concepts

These are discussed in more detail in the subsections dealing with the specific aspect.

6.1 Understanding shapes and their properties

Geometry begins with identifying and describing shapes. The primary classes are a stage at which children refine and extend their rough, intuitive understanding of shapes that they bring with them as pre-schoolers.

In the current practices, children first learn to name the basic 2-D and 3-D shapes, and then go on to learn the names of more and more complex shapes and terms related to them (sides, vertices etc.) as they move onwards from class I to V. However, it is very important to note that learning names and terms are merely stepping stones which will enable them to do more formal reasoning based work in geometry later.

In order to assess childrens' understanding of shapes, it is important to do a lot of exploratory activities with both 3-D and 2-D shapes, getting them to examine similarities and differences and talk about their various characteristics.

"Hmm... how can this be a triangle... it is too thin!"

Specifically, children at the primary level should be able to recognise, compare, sort, build and draw two and three-dimensional shapes. Before checking whether a child can identify and name the common 3-D and 2-D shapes formally, it would be necessary to see if she can do informal tasks using shapes involving comparing, finding shapes like a given one, etc. Later, children can be asked to identify common shapes like square, circle, rectangle, triangle, cubes and cones. Initially, young children recognise a shape by its overall appearance, without looking at their properties. For instance, they may say that a given figure is a triangle because "it is pointed at the top." There are various gaps in the understanding of shapes for children functioning at this level. Some of the more common ones could be:

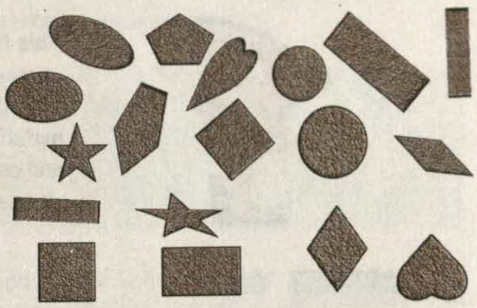
- Not recognising a shape on changing orientation. For instance, a child might recognise a triangle or square only in the 'regular' orientation – with the base horizontal and not in other orientations.
- Not recognising a shape if it is very small, very narrow etc.
- Thinking that only equilateral triangles (or those which look 'almost' like one) are triangles.

In some cases these misconceptions might persist even after the child is in high school. While assessing, it is important to keep these in mind and design graded tasks.

ACTIVITY 64

Conceptual area: Identifying 2-D and 3-D shapes.

Material: A collection of tiles, cardboard cut-outs of shapes, some of them should be the same. A sample set is shown.



Instructions: Take out one tile from the set, let us say, a regular hexagon. Ask the child to pick out all the other tiles that have *exactly the same shape* as that one.

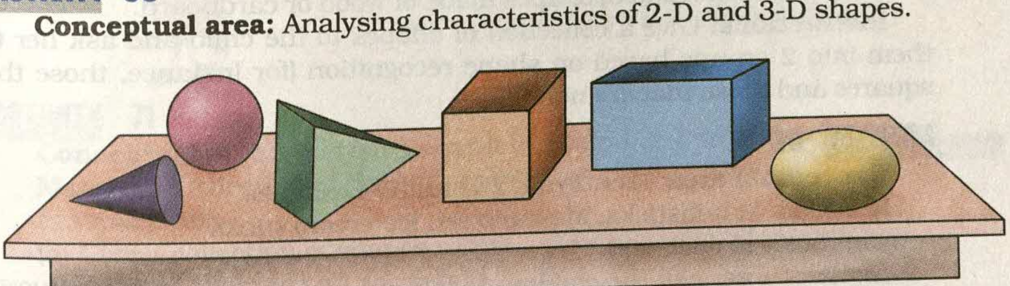
If the child is comfortably doing the initial level tasks, you can gradually increase the complexity – for instance, using a set that has tiles of the same shape but in different sizes or colours.

This task can help you assess if the child has formed initial notions about the concept of shape. See how she is comparing the shapes, does she place one over another to compare shapes, or does she do it visually. Also, check how careful she is about making sure no tile of the given shape is left out.

When other attributes like colour or size are introduced, see whether she is still doing the task confidently, or getting confused by these attributes.

ACTIVITY 65

Conceptual area: Analysing characteristics of 2-D and 3-D shapes.



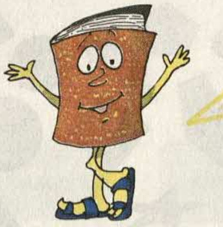
Material: Two identical sets of shapes (that is, each shape should be in duplicate), a box in which to keep one of the sets, a piece of cloth large enough to cover a shape fully.

For instance, one set could have a cone, a cube, a pyramid with a square base, a cuboid that is not a cube, a sphere, an egg shaped solid etc., and even some other shape. The second set should have duplicates of each of these shapes.

Instructions: Keep one set out of the two identical sets of shapes in a box. Display the second set on a table, so that the child can see them clearly and touch them. Now, choose any shape from the set in the box, and cover it fully with the cloth. (The children should not see which shape you are hiding!).

Now ask a child to come up, feel the shape under the cloth, and identify its 'match' from among the set of shapes kept on the table.

Later, when the child is familiar with the properties and names of shapes, the same game can be played without keeping an array for the child as a reference. In addition, shapes that are fairly close to each other, (for instance, a pyramid with a square base and one with a triangular base) may also be placed.



This task can help you assess the intuitive understanding of properties of shapes. If the child knows the names of the objects, she may just name them instead of pointing to the matching shape on the table. Notice how the child feels for and assesses various features and properties, for example - number of sides, vertices, equivalence of lengths etc.

ACTIVITY 66

Conceptual area: Identifying 2-D and 3-D shapes.

Material: Strips of paper, scissors, glue, chart paper or board and pencils

Instructions: Show a square cut-out to a child. Ask her to name the shape. If she names it correctly, rotate it around a little and ask what shape it is now. Note if she still identifies it as a square or not and ask her to give reasons for her answer. This activity can be done with all the shapes that the child knows of.

Identifying that a shape remains the same even when the orientation is changed develops later than simple shape recognition. If the child is able to answer the question correctly when the orientation is changed in front of her then probably you may want to give her a worksheet as given in Activity 69.

ACTIVITY 67

Conceptual area: Identifying 2-D and 3-D shapes.

Material: Collection of shapes made of wood or cardboard.

Instructions: Give a collection of shapes to the child and ask her to sort them into 2 groups based on shape recognition (for instance, those that are squares and those that are not).

ACTIVITY 68

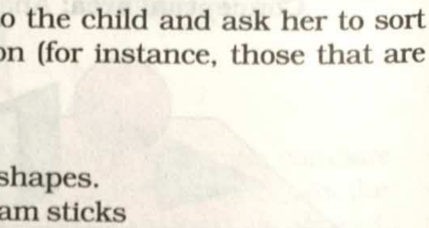
Conceptual area: Identifying 2-D and 3-D shapes.

Material: Matchsticks, Meccano set, ice-cream sticks with holes at each end.

Instructions: Ask the children to follow instructions as follows:

Make a triangle using these sticks.

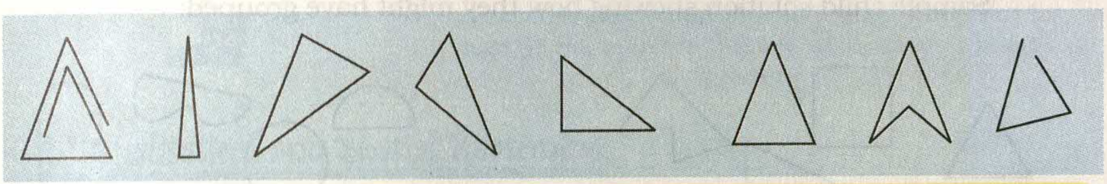
Make a rectangle using these sticks.



If a child makes a square when asked to make a rectangle, you would need to ask whether that is because the child is unclear with the vocabulary or has done it knowing that a square is also a rectangle.

ACTIVITY 69

Conceptual area: Identifying 2-D and 3-D shapes.
Material: Paper and pencil, or worksheet.
Instructions: Which of the following are triangles and why?



Has the child advanced to the level where she is identifying triangles (or any other class of shapes) by using their characteristic properties, rather than just visually? If she is at an initial level of understanding, she might identify Figure 1, or Figure 7 as triangles, saying they 'look like triangles'. Note whether she can identify 2, 3 or 4 as triangles, and what reasons she gives for her response.

ACTIVITY 70

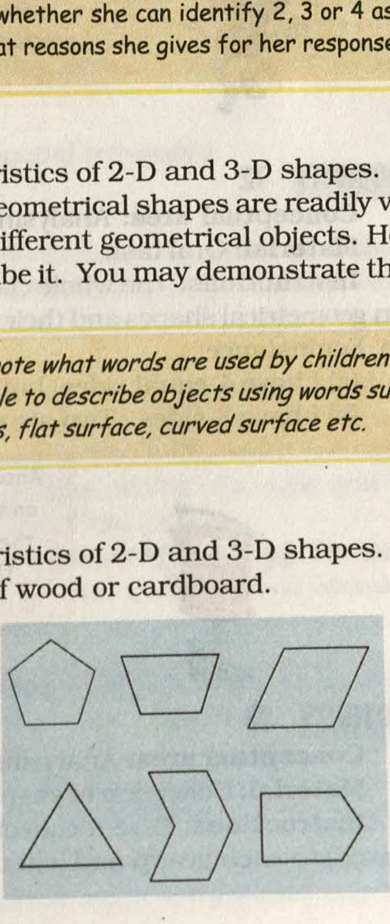
Conceptual area: Analysing characteristics of 2-D and 3-D shapes.
Material: Collection of objects where geometrical shapes are readily visible.
Instructions: Ask children to collect different geometrical objects. Hold up one object at a time and ask them to describe it. You may demonstrate the task once.

It is important to note what words are used by children and whether they are able to describe objects using words such as vertex, edges, flat surface, curved surface etc.

ACTIVITY 71

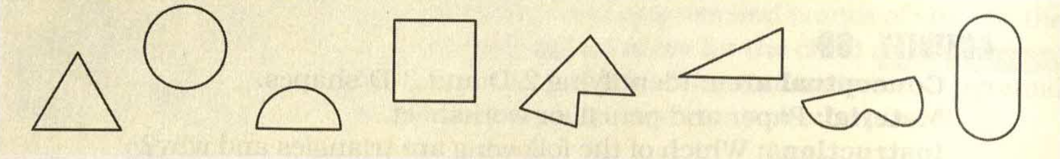
Conceptual area: Analysing characteristics of 2-D and 3-D shapes.
Material: Collection of shapes made of wood or cardboard.
Instructions

(a) Give a collection of shapes to the group and ask them to sort them into 2 groups based on some property of the shapes. For instance you could give the following set of shapes and ask them to sort the shapes into 2 groups – those which have *all* sides equal and those which don't.

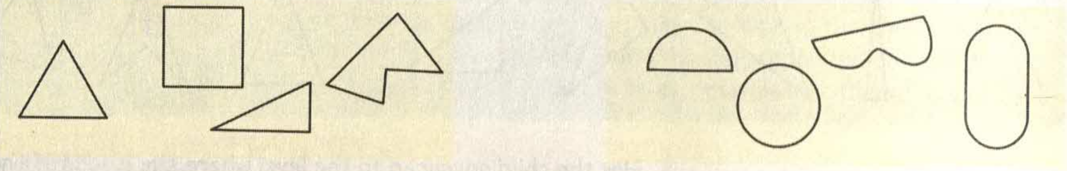


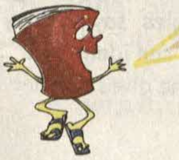
(b) A variation of this could be a group task where each group in turn gets a set of shapes, which they have to sort into two groups on any basis they want. Tell the children:

"Sort these shapes into 2 groups anyway you want, but give reasons for grouping them the way you did."



Sample child solution showing how they might have grouped:





It is important to ask the child about the basis that she used for classification into different groups as this would tell you about the awareness that the child has about different properties. Also, you would know whether the classification is correct or not.

ACTIVITY 72

Conceptual area: Analysing characteristics of 2-D and 3-D shapes.

Material: Oral task.

Instructions: The whole class can be posed with riddles like the ones given on geometrical shapes and their properties that children have had some exposure to. For instance:

(a) I have 4-equal sides, but my angles are not all equal. Who am I?

(b) I have 3-sides. Two are equal. Who am I?

Answering such questions would require children to focus on the properties and at the same time bring to mind all the shapes that they know of to check which one fulfils the condition. Depending on the questions asked the complexity of the question may be varied.

ACTIVITY 73

Conceptual area: Analysing characteristics of 2-D and 3-D shapes.

Material: Collection of shapes.

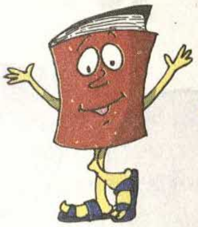
Instructions: Give a collection of shapes (either only 3-D, or only 2-D, or mixed) to each group and ask them to list those having a particular attribute.

For instance:

(a) Which of the following shapes have more than 3 vertices?

(b) Which have a curved face?

(c) Which of them have *at least* one pair of sides equal? Which of them have *exactly* one pair of sides equal?



Questions of this nature would help you analyse whether the children are able to identify, compare, and analyse attributes of two and three-dimensional shapes and have developed vocabulary to describe the attributes.

But remember, when you take up any activity of this kind for the whole class or group, you would need to be very observant and mobile to assess each child separately! This holds for most of the activities that are 'whole class' or 'group' activities.

6.2 Visualisation and Spatial Reasoning

Starting from hands-on experience in playing with shapes at an early age, children gradually need to develop the skills to manipulate them mentally, and visualise the results of enlarging, rotating, flipping, combining objects. They also develop the ability to predict what a shape would look like from different perspectives. Initially this is done by physically manipulating shapes and objects. Later, visualisation without these aids would be possible. Assessment should focus on checking if the child's abstract reasoning processes are becoming stronger. Some instances of tasks that can be used to assess the spatial reasoning skills of a child are described here.

ACTIVITY 74

Conceptual area: Visualisation and spatial reasoning.

Material: A collection of solid shapes.

Instructions

(a) Show a solid shape and ask the child to draw the shape that she would see if she were to look at the shape directly from top. Initially, start with simple shapes like a cylinder or a cube. Later more complex shapes like pyramids can be shown.

(b) For children who are already adept at the above, you can give more advanced tasks like putting one solid on top of another, and then asking them to draw or predict what the set-up will look like when seen from top, or from one side. For instance, put a cone on top of a cube and ask the children what it will look like when seen from top.

These tasks can sometimes be given in worksheets, in multiple choice format with options, to make sure that children are not getting stuck because they can't draw properly.

The difficulty of these tasks depends on the solid chosen, and even the direction from which it is seen. For instance, it may be easy for a child to figure out that a cylinder seen from top will look like a circle, but can she also figure out that when seen from the side, it looks like a rectangle? Or does she get confused because of the curved face of the cylinder?

Remember, some of these tasks could be a little tricky— you may need to check and hone your own Visualisation skills before giving these tasks to children!

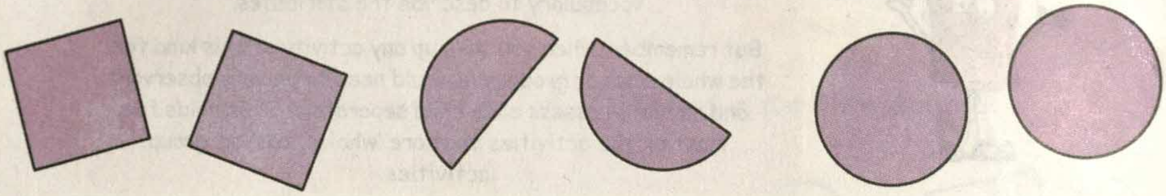
ACTIVITY 75

Conceptual area: Visualisation and spatial reasoning.

Material: Sets of 2-D or 3-D shapes – sets of wooden cubes, sets of tiles of different 2-D shapes, carboard cut-outs.

Instructions

(a) Show the child some tiles – let us say pairs of square tiles, semi-circular tiles, circular tiles etc, and ask: "Which pair of tiles can be put together to form a circle? Remember, no part of the tiles should overlap!"



(b) Show the child two square tiles and ask: "If we join these side to side, without any overlap, what shape will be formed?"

(c) Show the child two identical cubes. Ask: "If these were joined face-to-face, what shape would be formed?

(d) Take some identical square tiles. Show one of them and say: "I can form a square using exactly one square tile. If I wanted to form a larger square, how many of these square tiles will I need at least?". The task can be extended to predict for the next larger square.

Children demonstrating a more advanced understanding can also be asked to predict the number of edges, faces or vertices that the resulting shape would have.

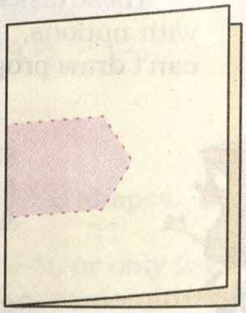
Observe whether the child needs to actually put together the tiles to name the new shape that will be formed. Gradually, she should be able to predict it correctly **without actually putting the tiles together** - maybe putting the tiles together just to verify that her prediction is right. Ultimately, the child should be able to predict and reason in abstract for example, in the last task, she should be able to mentally reason that the number of square tiles needed to form the next larger square will be 4.

ACTIVITY 76

Conceptual area: Visualisation and spatial reasoning.

Material: Paper for folding or cutting, pencil, paper or notebook to draw.

Instructions



(a) Fold a sheet of paper and mark any shape (for instance, the shape shown in the figure) on it. Now ask the child: "If the shape were cut out from the folded paper, what would it look like when opened?" (Children can be asked to draw it).

(b) A variation of the task could be to show the child a shape, and ask her to predict what shapes she would get if she were to cut the shape in a particular way. For instance, give her a square sheet, and ask her:

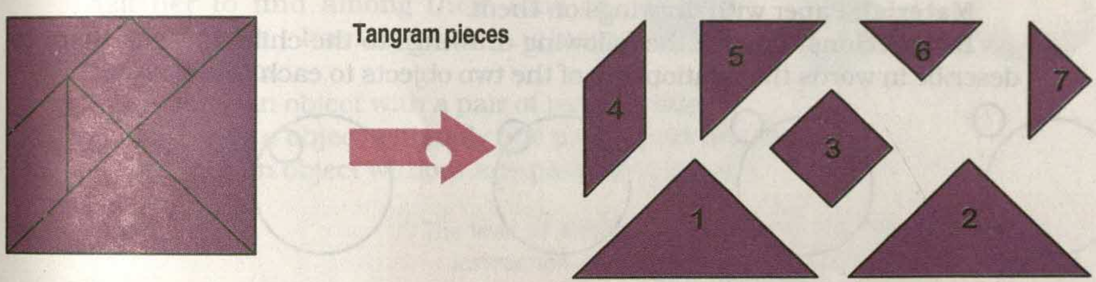
"What shapes would you get if you were to cut this along the diagonal?"

These kinds of tasks can help you check whether the child's ability to mentally compose and decompose shapes is being strengthened. Such imagination and skills could help children work more creatively in the areas of art or craft!

ACTIVITY 77

Conceptual area: Visualisation and spatial reasoning.

Material: Any set of tiles or cardboard cut-outs of different shapes; sets of tangram pieces with the pieces numbered as shown (A set of 7 puzzle pieces. You can make your own by cutting them out from a square sheet as shown here.)

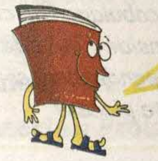


Instructions: Give the child the tangram pieces and ask him to make a particular shape with them. The tiles should not overlap. For example, you could say:

(a) Make a square, using pieces 1 and 2.
(b) Make a triangle using pieces 1 and 2.
(c) Make a rectangle, using pieces 3, 6 and 7.
(d) Make a triangle using pieces 5, 6 and 7.
(In these tasks, each of the the pieces mentioned need to be used)

Also, more advanced problems can be given, including challenges where a shape may not be possible to make with the tiles given.

(e) Make a triangle using pieces 4 and 5.



Always ask the children to justify their solution. For instance, if asked to make a square, ask her how she knows it is a 'square' or not a square. If she says it is not possible to make the required shape with the given tiles, ask her why it is not possible to make it.

A variation of the task could be to form groups, and give each group of children a similar set of 2 tiles. Each group is asked to try and form as many different shapes with them. This can help assess their Visualisation skills.

6.3 Understanding fundamental geometric concepts

For a primary school child, an introduction to formal work in geometry starts with basic concepts like what is a line, what is an angle, how one measures them and so on. She also needs to start understanding relations – like what are parallel lines, intersecting lines etc. Though it may seem that these concepts are very easy for children to understand, in reality, there can be several gaps in understanding even in these. For instance, many children might not be able to visualise a right angle in an orientation where the arms are not vertical or horizontal.

It is therefore, important to go beyond simply asking for 'definitions' while assessing children's understanding of these concepts. For instance – in the case of angles, this could be done by setting tasks that involve estimating, identifying, drawing and measuring angles in different orientations – rather than simply asking children to 'define' a right angle, acute angle or obtuse angle.

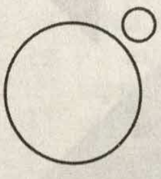
Some sample activities to assess understanding of basic geometric ideas and developing geometric vocabulary are given below. Note that there is a lot of overlap between the tasks related to shapes and these tasks.

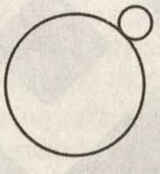
ACTIVITY 78

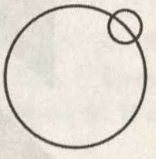
Conceptual area: Developing geometric vocabulary.

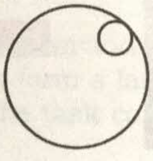
Material: Paper with drawings on them.

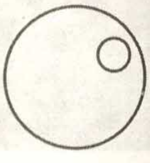
Instructions: Provide the following drawings to the child and ask them to describe in words the relationship of the two objects to each other.

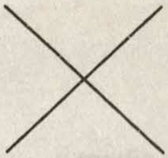

(a)

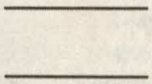

(b)

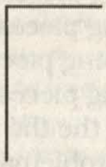

(c)

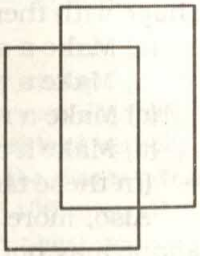

(d)


(f)


(g)


(h)


(i)


(j)

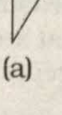
Note, in particular, the kind of vocabulary the child uses – is she using geometric terms like 'inside', 'outside', 'touching', 'intersecting', 'parallel', 'perpendicular' correctly and in appropriate places?

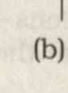
ACTIVITY 79

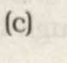
Conceptual area: Understanding and measuring angles.

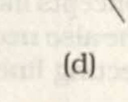
Material: Paper with drawings on them.

Instructions: Which one of the following angles is not a right angle?


(a)


(b)


(c)

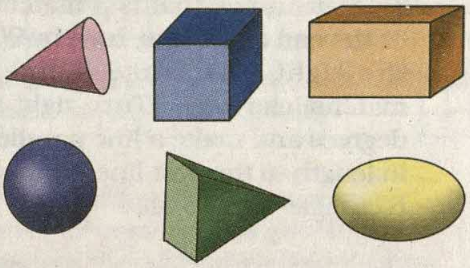

(d)

You need to ask the child about the basis she used to identify the right angles. It is important that the question has all kinds of options: right angles with horizontal arm, without horizontal arm, and acute or obtuse angles with or without horizontal arm.

ACTIVITY 80

Conceptual area: Developing geometric vocabulary.

Material: Set of 3-D or 2-D objects.



Instructions: Provide a set of 2 or 3-dimensional objects. Blindfold the child. Provide her with the set of objects and ask her to find among them (one or all) objects having some given characteristics:

(a) Find an object with a pair of parallel sides.
(b) Find an object with only one pair of parallel sides.
(c) Find an object without any parallel sides, etc.

The level of difficulty in an activity like this varies with the instruction and the language being used. Task (a) is less challenging for a child than Task (b) even if we assume that the child is competent enough to understand language like 'only one pair of parallel sides'. This is so because in Task (a) the child only needs to find one pair of parallel sides in a shape, whereas in (b), she has to also make sure that the shape has no other parallel sides.

The primary level is also the age where important geometric ideas like that of congruence, symmetry and coordinates are developed. The children can be exposed to (and later assessed on) a variety of exploratory activities related to these ideas. Tasks using a dot grid or paper folding also lend themselves well for developing and assessing these ideas.

ACTIVITY 81

Conceptual area: Understanding and measuring angles.

Material: Protractor, paper and pencil.

Instructions: Measure all the angles marked in the figure shown here.

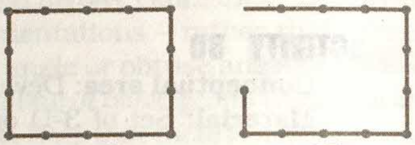
Usually, children are only asked to measure angles with horizontal arms. As a result, they may not develop facility with estimating and measuring angles in different orientations, and have difficulties later. Through this task, you will be able to check if the child can measure angles in any orientation comfortably and correctly. You can also check how well the child understands the concept of degree measure, by asking the child to estimate the measure of each angle before actually measuring it!

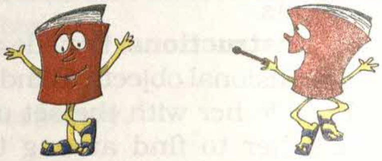
ACTIVITY 82

Conceptual area: Developing geometric vocabulary.

Material: Matchsticks and a partition.

Instructions: The two children are on either side of the partition and niether can see the other's work space. A shape is already made and provided to Child 1. And he/she gives instructions to Child 2 to reproduce the shape.





For instance, suppose Child 1 is giving instructions. She makes a figure in her side of the partition. Then, she may say to Child 2: "Make a line that is 3 matchsticks long. At the end of the line, turn by 90 degrees to the right and make a line that is 4 matchsticks long. Turn right again by 90 degrees and make a line parallel and equal in length to the first line. Now complete the rectangle" and so on.

Note whether Child 2 is able to make the figure or not depends on two things: the vocabulary used by Child 1 and the ability of Child 2 to follow descriptions/instructions, so both the children get assessed for different skills even as they are working in the same conceptual area.

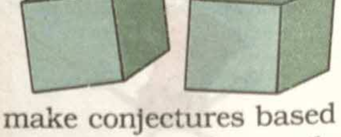
6.4 Developing logical reasoning ability

Developing analytical thinking is an integral part of learning geometry and needs to be a part of every geometry classroom, including classes 1 and 2. This has to take place through tasks that involve making predictions, drawing conclusions and justifying the same using appropriate vocabulary.

For very young children, the tasks can be situated in the ordinary shape sorting and identifying tasks they do. Many of the investigative tasks listed in the sections on shapes and visualisation are also tasks based on logic and justification. The main thing is for the teacher to remember to ask more and more of 'Why' questions and a lot less of the usual 'What' and 'Which' questions in class.

In most of the activities detailed in the earlier sections of this chapter, the analytical reasoning skill of the child can be assessed. Whenever the child is asked to justify her answer orally or on paper by writing reasons, drawing etc., the focus of assessment is on reasoning abilities. So in all these tasks, it is always good to remind oneself, that it is the **justification** that is more important, rather than just giving the correct answer.

ACTIVITY 83



Conceptual area: Analytical reasoning.

Material: Paper and pencil, or oral task.

Instructions: Children can be encouraged to make conjectures based on familiar shapes and terms related to them and informally prove or disprove them.

For instance:

Kavita has cut out two equilateral triangles. Will she be able to join them together (without any overlap) to form a square? Why or why not?

or

Ali has some identical cubes. He is trying to make a bigger cube by placing them together face to face. He claims that he has made a bigger cube using 2 unit cubes. Is that possible? Why or why not?

Solution of Child 1:	Solution of Child 2:
No, the new shape is not a cube.	No, I feel we can not make bigger cube with only two cubes because when he joins them some of the sides will be very long. Longer than other sides. And cubes must have all sides equal.

Remember, even for two children giving the same answer, there can be a lot of differences in the justification provided! In this case, both the children have given the same answer, but Child 1 has provided a clear justification for her prediction that the shape will not be a cube. It is important to provide every child the opportunity to articulate her line of reasoning.

Those who are not confident initially or provide reasoning that is logically or conceptually incorrect, can initially be guided. Later, it has to be seen if they are progressively getting more confident and skilled at reasoning..

6.5 Indicators

The indicators for assessing conceptual understanding and reasoning skills related to Shapes and Space for Classes I-V are presented in the following table. The teachers can develop their own indicators based on the syllabus.

	Conceptual area	Sample Activities	Proficiency Level		
			Basic	Intermediate	Advanced
Understanding shapes and their properties (6.1)	Identifying 2-D and 3-D shapes	64,66-69	Occasionally able to identify basic 2-D and 3-D shapes but might need support in more complex activities based on shape identification, sorting etc. *(Sample Activities: 64, 66-68).*	Mostly able to identify basic 2-D and 3-D shapes and demonstrates a reasonably good understanding of shapes. *(Sample Activities: 64, 66-68).*	Consistently able to identify basic 2-D and 3-D shapes, in any orientation. *(Sample Activities: 64, 66-68).* Demonstrates a deep understanding related to a shape or a class of shapes *(Sample Activity: 69)* Identifies shapes in the environment, and uses the names of shapes and other appropriate vocabulary related to these shapes in real-life situations.
	Analysing characteristics of 2-D and 3-D shapes	65, 70-73	Demonstrates an initial understanding of characteristics of shapes and vocabulary related to shapes, but may need prompting or support *(Sample Activities: 65,70-73).*	Demonstrates adequate understanding of characteristics of shapes and vocabulary related to shapes *(Sample Activities: 65,70-73).*	Demonstrates an advanced understanding of characteristics of shapes and vocabulary related to shapes and uses them appropriately in problems *(Sample Activities: 65,70-73).*
Visualisation and spatial reasoning (6.2)	Visualisation and spatial reasoning	74-77	Occasionally able to visualise 2-D representations of 3-D objects correctly or draw what an object would look like from different perspectives, but may always need to manipulate the object physically to do the task *(Sample Activity: 74).* May require support to perform the basic tasks involving spatial reasoning. *(Sample Activities: 75-77).*	Mostly able to visualise 2-D representations of 3-D objects correctly. May draw what an object would look like from different perspectives, but may need to manipulate the object physically to do the task (especially in complex cases) *(Sample Activity: 74).* Mostly able to perform the basic tasks involving spatial reasoning independently. *(Sample Activities: 75-77).* Sometimes uses spatial reasoning and Visualisation in problem-solving situations.	Consistently able to visualise 2-D representations of 3-D objects correctly, or predict what the object would look like from different perspectives – without having to manipulate the object physically. *(Sample Activity: 74).* Consistently able to perform complex tasks involving spatial reasoning independently. *(Sample Activities: 75-77).* Able to use spatial reasoning and Visualisation in problem-solving situations.
Understanding fundamental geometric concepts (6.3)	Developing geometric vocabulary	78, 80, 82	Occasionally able to relate the geometric concept like inside, outside, touching, intersecting, parallel, perpendicular, parallel sides etc. to the given figure and objects *(Sample Activities: 78, 80).* Occasionally able to relate the concept of right angle (90°), right turn or left turn to a given figure and draw it correctly *(Sample Activity: 82).*	Mostly able to relate the geometric concept like inside, outside, touching, intersecting, parallel, perpendicular, parallel sides etc. to the given figure and objects *(Sample Activities: 78, 80).* Mostly able to relate the concept of right angle (90°), right turn or left turn to a given figure and draw it correctly *(Sample Activity: 82).*	Consistently able to relate the geometric concept like inside, outside, touching, intersecting, parallel, perpendicular, parallel sides etc. to the given figure and objects *(Sample Activities: 78, 80).* Consistently able to relate the concept of right angle (90°), right turn or left turn to a given figure and draw it correctly *(Sample Activity: 82).*

Developing logical reasoning ability (6.4)

Conceptual area	Sample Activities	Proficiency Level		
		Basic	Intermediate	Advanced
Understanding and measuring angles	79, 81	Has an initial level understanding of the concept of an angle, and tries to recognise right angles in the environment. *(Sample Activity: 79).* Occasionally able to estimate the size of angles, measure them, or classify them (as right, acute, obtuse), but may need support to do such tasks *(Sample Activity: 81).*	Understands the meaning of an 'angle' and is mostly able to recognise right angles in the environment or in classroom tasks, but might have problems when the angle is not shown in regular orientation *(Sample Activity: 79).* Mostly able to estimate the size of angles, measure them, or classify them (as right, acute, obtuse), but might be confused in non-standard tasks involving these skills *(Sample Activity: 81).*	Understands the meaning of an 'angle' and is consistently able to recognise right angles in the environment or in classroom tasks, even when the angle is not shown in regular orientation *(Sample Activity: 79).* Consistently able to estimate the size of angles, measure them, or classify them (as right, acute ,obtuse), even in non-standard tasks involving these skills *(Sample Activity: 81).* Uses terms and concepts related to angles, angle estimation or measurement appropriately for problem-solving in the classroom or in life situations.
Analytical reasoning	83	Demonstrates initial level attempts at reasoning based tasks, but may require support in analysing the problem, or reasoning in a stepwise manner towards solving it *(Sample Activity: 83).*	Demonstrates an intermediate level of skill at reasoning based tasks – may occasionally analyse the problem independently or reason in a stepwise manner towards solving it *(Sample Activity: 83).*	Demonstrates an advanced level of skill at reasoning based tasks – analyses the problem independently and reasons in a stepwise manner towards solving it. Is also able to provide logically sound justifications for her every step in the solution *(Sample Activity: 83).*

7

Measurement

Measurement is an area of mathematics of immense practical importance. The concepts in measurement lie in doing and in relating to acts of measurement in the real world. Basically, measurement involves comparison of physical attributes of an object or being - length, area, volume, time, weight, and angle are some of the basic attributes that are part of the learning of children in school.

Learning about measurement involves - the concepts, the related vocabulary, skills of using measuring instruments and familiarity with units of measurement. The various dimensions of learning measurement are discussed below.

- Intuitive grasp of the attribute and concept of comparison.
- Using an intermediary to compare two things (where direct comparison is not possible).
- Understanding the idea of iteration of a unit.
- Knowing and using standard units.
- Measurement of complex shapes (involving combination of measurements)
- Knowledge and use of instruments for measuring.

The present approach to assessment of concepts of measurement also tends towards using materials that need to be measured as well as measuring instruments in a problem solving mode.

7.1 Intuitive grasp of attribute and concept of comparison

The child begins learning about measurement through an informal understanding of the attributes of length, weight, volume and area. At this stage, she needs to learn precise comparative words to use for different attributes – long, longer etc. for length, heavy, heavier, lighter etc. for weight and so on. Assessment at this stage should check for understanding and using these terms appropriately, and also the skill of making direct comparisons of these attributes.

ACTIVITY 84

Conceptual area: Intuitive understanding of length, area, weight or volume.

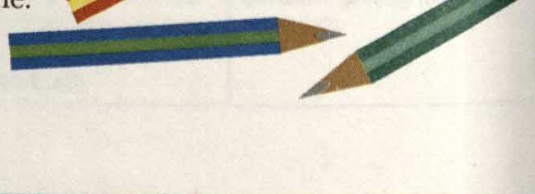
Material: Pencils or sticks or straws of different lengths.

Instructions: Give the child a set of pencils or sticks or straws of different lengths. Pick one up from the set and ask:

Find me a pencil longer than this one.

Find me a pencil shorter than this one.

Find me a pencil as long as this one.



By observing the child's actions and answer in this task you can tell if the child has an intuitive understanding of the concept. Also, you can check if the technique of comparison is in place or not, that is, does she align the ends of the pencils properly etc.

ACTIVITY 85

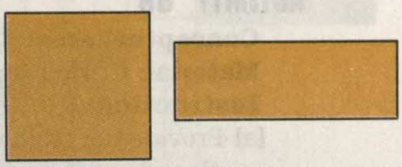
Conceptual area: Intuitive understanding of length, area, weight or volume.

Material: pencils or sticks of different lengths, irregular shapes made out of card sheet, empty glasses or tiffin-boxes or any hollow shapes of comparable as well as different sizes, sand, water or saw-dust.

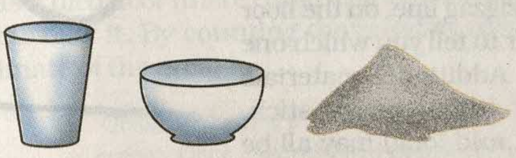
Instructions:

(a) Give the child two pencils, and ask: "Which one is longer and which one is shorter?"

(b) Give a pair of flat shapes and ask: "which is bigger"? Observe how the child superimposes one over the other to compare area, whether she compensates for bits that may stick out etc.



(c) Provide hollow shapes (like cups, bowls) and water/sand/saw dust and ask to find out which has the larger volume/smaller volume.



For concept of weight two objects could be given and the child could be asked, "Which is heavier? Which is lighter?"

ACTIVITY 86

Conceptual area: Intuitive understanding of length, area, weight or volume.

Material: Pencils, sticks.

Instructions: Give the child a set of pencils / sticks of different lengths. Ask them to arrange these pencils from longest to shortest on the table/ground.

Observe how the child arranges these. Does she complete these by trial and error or does she have a method in place to pick out and place the next one?

Now give her another pencil, and now ask her to put it into the set. Observe how she identifies where the pencil should go in the series.

ACTIVITY 87

Conceptual area: Informal understanding of angles.

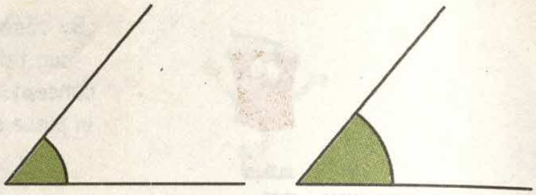
Material: Different objects available in the environment.

Instructions:

(a) Children should be able to identify objects with edges and the 'angle' made at the joining of these edges.

Further they could be asked questions as to which angle is larger than the other. This would also give an idea if the child has a correct 'sense' of angle or has she confused it with space between the two arms.

(b) Draw two equal angles. Draw the arc of the angle at two different positions in the two angles – one near the vertex and one relatively away from the vertex. Ask the child, which angle is greater and why?



(c) You can position two fingers at a particular angle and ask the child what angle it is. You can follow this question with "if a person with longer fingers were to make the same position would the angle be larger?"

Through the above activities one can understand if the child has an intuitive understanding of the concept of angle. One can also clarify if the child has confused the concept of angle with the distance between the two arms.

ACTIVITY 88

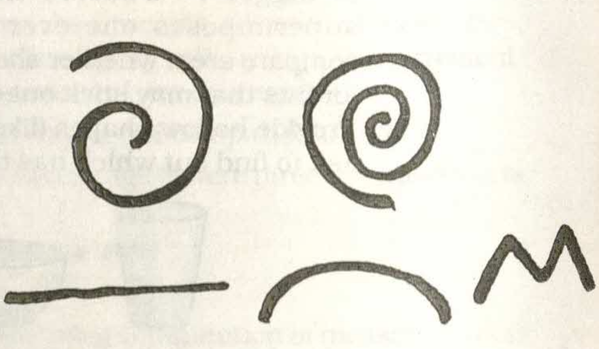
Conceptual area: Understanding length measurement for curved shapes.

Material: Curled ropes, paper, pencil.

Instructions

(a) Provide the child with two ropes, placed curled up and ask her which one is longer.

(b) Draw a straight line, a curved line, and a zigzag line, on the floor and ask her to tell you which one is longest. Additional materials like longer stick, shorter stick, rope (short and long) may all be provided.



This activity can help you find out if there are any children who have the misconception that only straight lines or objects have length, or that the length of curved objects can't be measured.

7.2 Using an intermediary object to make comparisons

Once an understanding of the attribute is in place, the child learns to make comparisons in situations when the objects can not be compared directly, or direct comparison may not be accurate. Here, the assessment should focus on what kind of strategies the child formulates for measuring such objects – whether she is using an intermediary object – like a rope, her foot, a stick etc, and also, how appropriate the choice of the intermediary is.

ACTIVITY 89

Conceptual area: Concept of using an intermediary object for measurement.

Material: Sticks of comparable (but not same) lengths, pieces of rope/thread, matchsticks, sticks of different lengths.

Instructions: Provide the child with two sticks that are close (but not necessarily equal) in length and fixed at a distance from each other. Provide additional materials such as sticks, longer and shorter, rope, etc. Ask her to compare the two and tell you which is longer, and observe how she does it.

Instead of comparing two fixed sticks, the child can also be asked to compare any two things that are of comparable lengths, but can't be compared directly.

For instance: the height of the classroom door and the door of the child's room.

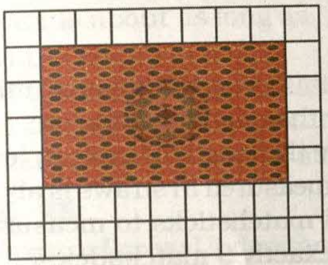
If she uses a longer stick, to make the measure and compare, next ask her to make the measurement using the shorter stick and whether she is able to iterate and count. Also check if she is using the same stick or different sticks for comparing the two given sticks. This would tell us if the initial idea of a unit and iteration are forming.

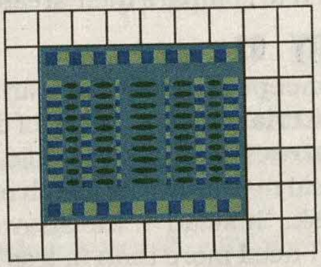
ACTIVITY 90

Conceptual area: Concept of using 'units' for measurement.

Material: Carpet/tiles painted/drawn on square grid.

Instructions: Provide the child with the picture here and ask, which carpet is larger?





If the picture has a tiled floor (there could also be floor tiled with say triangular tiles and two carpets over it. By counting the number of tiles covered the child would have an estimate of the area.

Observe the way in which pupils decide the larger or smaller carpet. Once children know formal units, ask them to find the area if area of a single square is given. Assign different area to the same grid-square and check whether children change their answer. It is important that the area depends on the unit chosen. For example, if the area of a single square is given to be 1 cm^2, the total area of the carpet on the left side would be 35 cm^2. On the other hand, if a single square of the grid is marked as 4 cm^2, the area of the same carpet would be 140 cm^2.

7.3 Understanding the idea of iteration of units

Along with the understanding that in most measurement situations an intermediary object or a measurement instrument has to be used, comes the understanding of using 'units'. The unit has to be used 'iteratively' – that is, by repeating it as many times as needed to measure the given object.

At this stage, some important aspects to assess are how well the child is using informal units for measuring, and whether he has grasped important ideas like – the same attribute of an object can be measured using different 'units', and the relation between the size of the unit and the measure of the object.

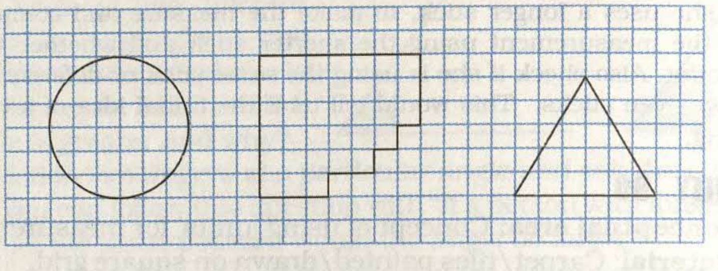
ACTIVITY 91

Conceptual area: Comparison of attributes by iteration of units.

Material: Shape cut-outs and grid paper.

Instructions: Provide shapes (regular/irregular/polygonal/circle etc) drawn on triangular grid and ask to compare areas: Which has the greatest area? Which has the smallest area?

Shapes could also be drawn on a square grid for the same exercise.



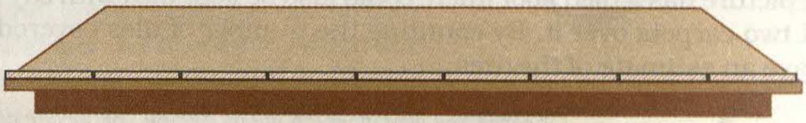
A variation of this could be to draw two identical shapes on transparent paper and place them on the square grid paper in different orientations and ask children to compare their areas.

ACTIVITY 92

Conceptual area: Comparison of attributes by iteration of units.

Material: Straws, sticks of different lenths, match-sticks, etc.

Instructions: Ask the child to measure the length of table in terms of straw. The child may say: "The length of the table measured in straws is about 9 straws." Now ask: "Instead of the straws, if I used matchsticks to measure the length, would I need more than 9, less than 9 or exactly 9 matchsticks?



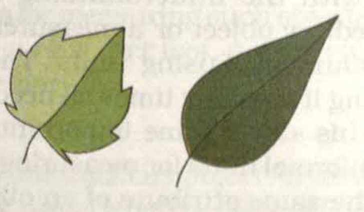
This Activity can help you check if the child understands that the number of iterations is related inversely to the unit size-that is, the bigger the unit with which you are measuring, the fewer the numbers needed. It is likely that at the beginning stages children think that bigger units mean more of them will be needed.

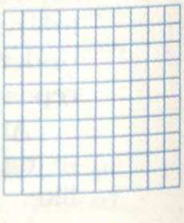
ACTIVITY 93

Conceptual area: Comparison of attributes by iteration of units.

Material: Some flat objects – like leaves or paper cut-outs, grid paper or graph paper.

Instructions: Give the child two leaves of irregular shape and ask her to compare the areas and tell you which is larger. Along with the other material around such as rulers, string, also let there be square and triangular grid sheets.





Note if the child intuitively uses counting squares to compare the areas. Also, observe the way in which incomplete squares are counted/omitted/rounded off. Try to talk to the child to find out the reasons of counting, omitting or rounding off incomplete squares. Check if children can decide which is larger or smaller shape without using grid or graph-paper.

7.4 Knowledge and use of standard measures

Appreciating the need for standard units and being familiar with the use of these are also important for the primary school child. Students need to learn and be assessed on whether they know the commonly used units, the relationship between different units to measure the same attribute (like – m and cm) and can use these units appropriately to measure or estimate.

ACTIVITY 94

Conceptual area: Understanding and using standard units of measurement.
Material: Oral task.
Instructions: If the child has knowledge of standard units, ask

A metre is about as long as

(a) the length of my leg
(b) the length of my book
(c) the length of my foot

Similarly, children can be asked to estimate other standard measures.

ACTIVITY 95

Conceptual area: Understanding and using standard units of measurement.
Material: Oral task.
Instructions: Name an animal which can be about 5 cm long.

Ask her how did she conclude the length of the animal or insect is about 5 cm? Discuss with her that if she were to actually measure length of a creature, how would she go about doing that.

ACTIVITY 96

Conceptual area: Estimation of time.
Material: This is a discussion type activity. You do not need any material for this.
Instructions: Ask children questions on how long does it take to do different things. Ask them to make a table and write their own estimates.

Eat dinner	
Blink	
Clap once	Less than a second
Clap fifteen times	
Writing your full name	
Count from 1001 to 1060	

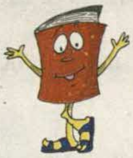
Let children fill both the columns on their own. Let them think of their favourite activities and estimate time. Include Activities like time required for a particular seed to germinate or a person to recover from common cold, etc.

ACTIVITY 97

Conceptual area: Estimation of length.

Material: Objects of different lengths e.g. pen, long stick, notebook, school-bag, etc.

Instructions: Provide different objects to children and ask them to assess their length. Ask children to measure each other's height, waist-size, etc. Also ask them to first guess and then measure length and breadth of the room, width of the door or window etc.



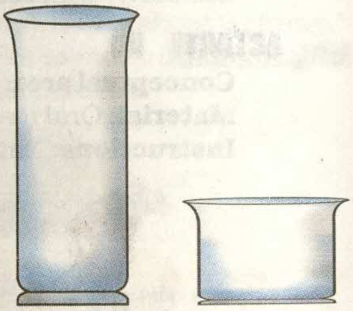
It is important that the teacher herself knows the lengths of objects that she intends to give to children, and even hones her own estimation skills. If the child's estimate is too far off, ask relevant questions to help the child make a better estimate.

ACTIVITY 98

Conceptual area: Measurement of volume

Material: Two jars of different shapes. Some water in one of them

Instructions: Pour the water from a jar, say a tall, thin one, into a shorter one and ask the child what happens to the volume of water.



This activity will help you determine how well-formed the concept of volume is in the child's mind. Does she realise that the total volume of liquid is being conserved, or does she think the volume of water changes when it is poured into a differently shaped vessel?

ACTIVITY 99

Conceptual area: Measurement of weight.

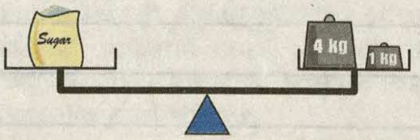
Material: Weighing balance (grocer's balance, not electronic one), rice, wheat, sand, stones, fruits, etc.

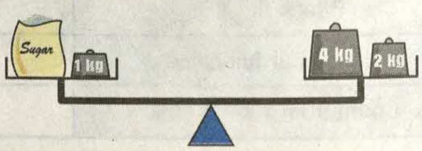
Instructions: Using standard weights to measure—ask children to weigh out one kilogram of rice or sand, and 1.5 kilogram, 1 kilogram 100 grams etc.

Later, exercises could also include:

Find me an object about twice as heavy as this one, etc...

"If I had weights of 1kg, 2kg and 4kg only, can I weigh 6kg by using these weights?" If the child finds this easy, you can give problems like, Can you show how to weigh 5kg using these weights in more than one way?





ACTIVITY 100

Conceptual area: Understanding and using standard units of measurement.

Material: Oral task or in worksheet.

Instructions: Give children problem situations like – Anita, Sunita and Vinita ran the same race. Anita took 3 minutes and 5 seconds, Sunita took 4 minutes and 2 seconds, Vinita took 3 minutes and 7 seconds. Who won the race?

ACTIVITY 101

Conceptual area: Estimation of area.

Material: Black board and chalks or notebook and pen/pencil.

Instructions: Draw a box about 5 cm by 6 cm by 4 cm. Ask the child: "How many square centimetres do you think will fit in at the top of the box?"

Answers to the question will help you to understand if children have developed a sense of the size of the unit.

You may draw different shapes or draw the same shape and size with different orientation and repeat the question. You may also ask questions like "How many square metres will cover the floor of this classroom?"

ACTIVITY 102

Conceptual area: Understanding and using standard units of measurement.

Material: This can be an oral or a written activity. Children may use paper-pencil if they want but you don't need any other material.

Instructions: Ask questions like: "It takes about 4 hours by train from Bangalore to Mysore. If the train starts from Bangalore at 11 am, at what time will it reach Mysore?"

Be prepared with a set of such questions with varied degree of difficulty. If the child finds it difficult to answer a particular question, give her a simpler question and vice versa.

ACTIVITY 103

Conceptual area: Understanding and using standard units of measurement.

Material: Oral task.

Instructions: Give the child situations like: "After measuring the length of the table, the carpenter wrote down 54 in his note book and forgot to mention the units. Which of these units do you think is the right one? Cm, m, or km?"

If you think that the child is not choosing proper unit, check if she understands the difference between units. Also ask relevant questions like – Is your ruler 15 cm or 15 m, or, is your weight 22 kg or 22 gram etc. – to take her to the choice of the most appropriate unit.

ACTIVITY 104

Conceptual area: Measurement of time.

Material: Calendar.

Instructions

(a) Ask children questions like: "How many weeks are there from 7 January to 24 February?" Or "The summer holidays start on May 15th. Today is March 23. How many more days to go before the summer holidays start?"

(b) Show the calendar of a month to the child, say for example, calendar of March 2008. Ask her to make the calendar for the month of April 2008 or February 2008. Also ask questions like: "On what day of the week will April 15th fall"?

MARCH 2008

MON	TUE	WED	THU	FRI	SAT	SUN
					1	2
3	4	5	6	7	8	9
10	11	12	13	14	15	16
17	18	19	20	21	22	23
24	25	26	27	28	29	30
31						

ACTIVITY 105

Conceptual area: Understanding and using standard units of measurement.

Material: If you are doing this exercise orally, you do not need any material; otherwise you need weights and weighing scales.

Instructions: I have only 500 grams weight. How shall I measure one kilo? Two kilogram? 250 gram? (Can be done practically or orally, depending on how well the children can reason).

7.5 Measurement of complex shapes and use of formulae

Once clear with the basics of measurement, children are also expected to develop, know and use formulae to compute area or perimeter of certain basic shapes like rectangles, and be able to handle more complex problems based on these ideas.

ACTIVITY 106

Conceptual area: Computing area and perimeter

Material: Several objects like compass box, notebook, textbook, circular dish, CD of computer, foot-scale, etc.

Instructions: Present the child with any of the objects and ask which has the longest perimeter and shortest area, or equal perimeter and unequal areas etc.

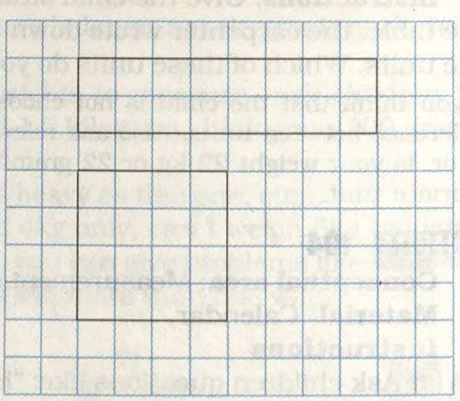
This activity is meant to understand if children are making out the difference between area and perimeter. In case of any three dimensional object, you will have to specify the surface whose perimeter and area are to be measured. Let the child guess the perimeter/area and then actually compute by doing measurement. Ask questions related to the difference between the guessed and measured values.

ACTIVITY 107

Conceptual area: Computing area and perimeter.

Material: Drawing sheet or worksheet, pencil.

Instructions: Draw two objects with same area but different perimeters (such as a rectangle of 8 × 2 and a square of 4 cm) on a square grid and ask: 'Which has the greater perimeter?'



Children tend to confuse perimeter (or circumference) with area. Asking children to provide the perimeter of objects would tell us if they have grasped the concept. Also, note the example has two figures whose area is the same. Further questions based on this could also be asked to check for deeper understanding – "Is it possible for two figures with the same area to have different area? Two figures with the same perimeter to have the same area? Can you draw and show?"

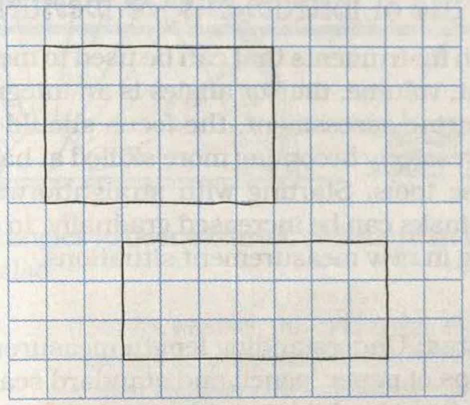
ACTIVITY 108

Conceptual area: Computing area and perimeter.

Material: Paper-pencil, cm-square grid paper.

Instructions: Give a blank grid paper to the child and ask her to draw a square with a given perimeter, say 8 cm. Or to draw two different rectangles both with perimeter 20 cm, etc.

Sample child solution:



ACTIVITY 109

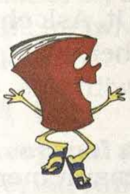
Conceptual area: Complex reasoning with area and perimeter.

Material: Cm-square grid or graph paper.

Instructions: Provide square grid or graph paper and ask the child to draw

(a) a square of area 4 cm² on it

(b) a rectangle of area 12 cm² on it



While the child draws the shape, observe carefully and ask questions relevant to the strategy. For instance, a child may draw a rectangle of base 6 cm and height 2 cm to get area 12 cm², while some other child may draw a rectangle of base 4 cm and height 3 cm to get rectangle of the same area. Also ask them how they arrived at the number pair 6 and 2, or 4 and 3.

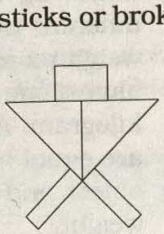
ACTIVITY 110

Conceptual area: Complex reasoning with area and perimeter.

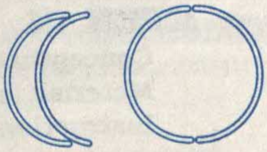
Material: Drawings of complex shapes (made up of various standard shapes as illustrated in the following figure), or shapes made of matchsticks or broken bangles.

Instructions

(a) Provide a complex shape made of sub-shapes and ask questions about the total area of larger shape made of smaller ones. (The child is allowed to measure the sides of the component shapes if she wants).



(b) Make a circular shape by joining two pieces of bangle. Trace it out on paper. Now, using the same two pieces, form a concave shape and trace that out too. Now ask the child to compare the perimeter of the two shapes. Ask them to justify their answer.



Later, it can be done by drawing different shapes on paper and asking them to compare the perimeter.

Observe the strategy followed by the child to answer the perimeter problem. If the child asks for thread or thin rope, provide the same.

7.6 Knowledge and use of instruments for measuring

Being familiar with instruments that can be used to measure different attributes like length, weight, volume, time or angles is an integral part of learning about measurement. During assessment, the focus should be on checking whether children are progressively becoming more skilled at handling tasks that require them to use these tools. Starting with straightforward tasks, the degree of complexity of the tasks can be increased gradually, to see if the child can adapt his understanding in new measurement situations.

ACTIVITY 111

Conceptual area: Understanding length measurement and using ruler.

Material: Strips of paper, pencil and standard scale or ruler.

Instructions: Provide children with a strip of paper, and pencil, and ask them to make their own ruler. Observe what unit they choose, do they make markings at equal intervals, do they mark at the corner of the paper, etc.

ACTIVITY 112

Conceptual area: Understanding length measurement and using ruler.

Material: Ruler with centimetre marking, paper.

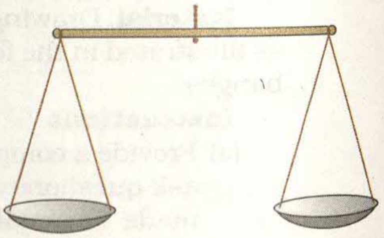
Instructions: Arrange a ruler with cm marking and ask them to measure thickness of a very thin object, say a paper or a hair with it. Ask children to give reasons for their inability to measure it accurately. Further, ask questions like what is the smallest length you can measure accurately with this, etc.

ACTIVITY 113

Conceptual area: Understanding weight (mass) measurement and using beam balance.

Material: Weighing scales, weights and some objects to weigh.

Instructions: Give a beam balance to the child and 1 kg weight. Provide an array of objects of different weights. Ask her to sort out those that weigh more than one kilogram, those that weigh almost one kilo, and those that weigh less than one kilogram. Ask her to find a pair of objects which are equal in weight to each other, or give her one object and ask her to find another of the same weight.



ACTIVITY 114

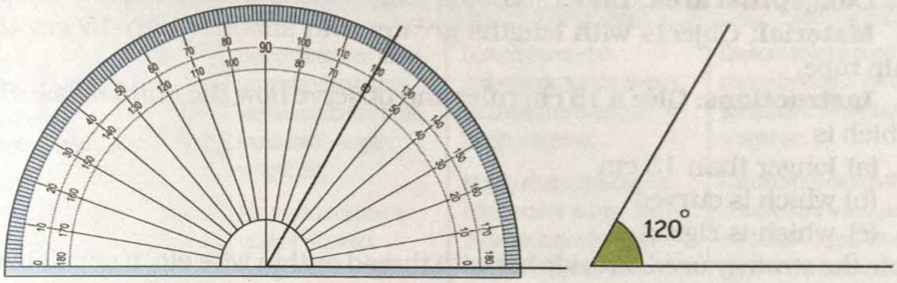
Conceptual area: Understanding angle measurement and using protractor.

Material: Protractor, paper and pencil or worksheet.

Instructions: Provide children with drawings of angles and a protractor. Ask her to measure the angles in the drawing. Start with simple angles, with at least one arm horizontal, and the arms of the angle extending beyond the circumference of the protractor.

Task: Draw an angle of 120^0.

Sample child response:



Instead of giving angles to be measured, children can be asked to draw angles of a given measure also.

Examine whether the child is able to place the protractor for correct zero alignment and count from zero. A common error that children make while using the protractor is to count from the 'wrong side' – for instance, drawing an angle of 120° degrees as shown here. Ask such children to always draw a rough 'estimate' of the angle first and then the actual figure using the protractor, and compare the two. See whether their errors reduce with this strategy.

(b) Later, give the same task, but with angles that are drawn in different orientations, or, angles that may have arms that are shorter than the radius of the protractor (where the child has to either extend arms or examine for alignment within).

ACTIVITY 115

Conceptual area: Understanding length measurement and using ruler.

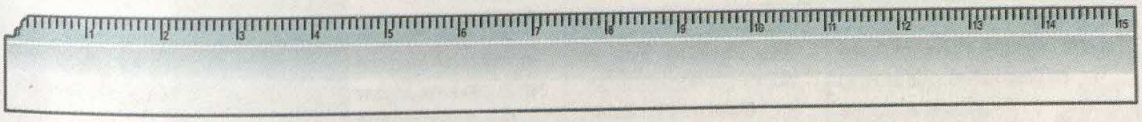
Material: Ruler, pencil, paper.

Instructions: Provide ruler and pencil and paper to the child and ask her to draw a line-segment that is more than 12 cm long or draw a line-segment that is less than 5 cm long.

ACTIVITY 116

Conceptual area: Understanding length measurement and using ruler.

Material: Ruler with broken edge (ruler without zero mark), different objects like book, notebook, pencil, etc.



Instructions: Provide children with a ruler with a broken edge (procure a ruler with zero mark broken or improvise a ruler) and ask them to measure length and breadth of different objects using the broken ruler.

If you feel that the child has not done proper measurement, provide her with a good ruler and ask to measure the same object. Ask her to reason out the difference between two measurements.

ACTIVITY 117

Conceptual area: Understanding length measurement and using ruler.

Material: Objects with lengths greater and smaller than 15 cm thread or thin rope.

Instructions: Give a 15 cm ruler and observe how the child measures object which is

(a) longer than 15 cm

(b) which is curved

(c) which is zigzag

Note the strategy used. Provide her with thread or thin wire etc. if she demands it.

ACTIVITY 118

Conceptual area: Understanding angle measurement.

Material: Clock with minute and hour hand.

Instructions: Show the clock with some time, say, 3 hrs. Ask children about the angle between the two hands. Repeat the question for different times. After initially estimating, the childs could actually be asked to measure to check how close their estimate was.

7.7 Indicators

The indicators or markers of quality for assessment of achievement in Measurement for Classes I-V are presented in the box. The teachers can develop their own indicators based on the syllabus.

	Conceptual area	Sample Activities	Proficiency Level		
			Basic	Intermediate	Advanced
Intuitive grasp of attribute and concept of comparison [7.1]	Intuitive understanding of length, area, weight or volume	84-86	Demonstrates an initial understanding of the various attributes to be measured – length, weight etc. May need support to relate the correct comparative words used for these attributes (longer, heavier etc). May get confused between various attributes *(Sample Activities: 84-86).*	Demonstrates a fair understanding of the various attributes to be measured – length, weight etc. Mostly relates the correct comparative words used for these attributes (longer, heavier etc.) correctly. May sometimes get confused between various attributes or use comparative words that are not precise - like 'bigger' for both length and area *(Sample Activities: 84-86).*	Demonstrates a good understanding of the various attributes to be measured – length, weight etc. Consistently relates the correct comparative words used for these attributes (longer, heavier etc) correctly and uses the precise comparative words in any context –without getting confused between various attributes *(Sample Activities: 84-86).*
	Informal understanding of angles	87	Demonstrates an initial understanding of the concept of an 'angle'. May need support most of the time to distinguish angle measure from other attributes, say length or area. Occasionally able to compare angles informally *(Sample Activity: 87).*	Demonstrates a fair understanding of the concept of an 'angle'. May show confusion about angle measure in more complex tasks. Mostly able to compare angles informally *(Sample Activity: 87).*	Demonstrates a good understanding of the concept of an 'angle'. Does not get confused about angle measure – even in more complex tasks, for example, understanding that angle measure does not change by just changing the length of the arms. Consistently able to compare angles informally *(Sample Activity: 87).*
	Understanding length measurement for curved shapes	88	May demonstrate the misconception that curved objects cannot have length or that the length can not be measured*(Sample Activity: 88).*	Demonstrates some understanding of the fact that curved objects can also have length. But may have some misconceptions still – for example, thinking that the curved object has length only after it is straightened *(Sample Activity: 88).*	Demonstrates complete understanding of the fact that curved objects can also have length. Consistently able to compare the length of two or more curved objects which can be straightened (like ropes) by direct comparison. Thinks of strategies to measure or estimate the length of curved lines, curved faces of objects etc. that can not be straightened *(Sample Activity: 88).*
Using an intermediary object to make comparisons [7.2]	Concept of using an intermediary object for measurement	89-90	Requires support to compare the length (or weight, area etc) of two or more objects that can't be compared directly. Concept of using an intermediary object for measurement not yet developed *(Sample Activities: 89-90).*	Sometimes able to compare the length (or weight, area etc) of two or more objects that can't be compared directly. Concept of using an intermediary object for measurement beginning to develop, but may still have trouble with the tasks – for example, in selecting an appropriate intermediary object *(Sample Activities: 89-90).*	Consistently able to compare the length (or weight, area etc) of two or more objects that can't be compared directly. Concept of using an intermediary object for measurement well developed. Consistently able to select and use an appropriate intermediary object for measurement in such tasks *(Sample Activities: 89-90).*

	Conceptual area	Sample Activities	Proficiency Level		
			Basic	Intermediate	Advanced
Understanding the idea of iteration of units (7.3)	Comparison of attributes by iteration of units.	91-93	Demonstrates an initial understanding of the concept of using 'units' iteratively for measurement. Requires support in such tasks *(Sample Activities: 91-93).*	Demonstrates a fair understanding of the concept of using 'units' iteratively for measurement *(Sample Activities: 91-93).*	Demonstrates a good understanding of the concept of using 'units' iteratively for measurement – including an understanding of the inverse relation between unit size and the measure of the object . Consistently able to choose an appropriate intermediary object (or informal unit) to find and express the length (or area etc.) of a given object. *(Sample Activities: 91-93).*
Knowledge and use of standard measures (7.4)	Understanding and using standard units of measurement.	94-95, 100, 102-103, 105	Demonstrates some awareness and understanding of the standard units of length, weight and volume. *(Sample Activity: 94).* May need support to recall the relationship between units (for instance m and cm) and to handle problems requiring conversion of units *(Sample Activities: 95,100,102-103,105).*	Demonstrates awareness and understanding of the standard units of length, weight and volume. *(Sample Activity: 94).* Knows the relationship between units (for instance m and cm) and is sometimes able to convert units as required. At times, may make errors while converting, or get confused – especially while converting from smaller unit to larger (for example, writing 48 cm as 4800 m or 4.8 m) *(Sample Activities: 95,100,102-103,105).*	Demonstrates full awareness and understanding of the standard units of length, weight and volume. Knows which unit is used to measure what, and has a good estimate of how long a cm or m is, how heavy 1 kg is etc. *(Sample Activity: 94).* Knows the relationship between units (for instance m and cm) and uses them appropriately – that is, converting from one to another if needed *(Sample Activities: 95,100,102-103,105).* Demonstrates a complete understanding of the importance of standard units and selects and uses the appropriate units in any situation in the classroom or outside.
	Measurement and estimation of length, weight, volume, time or area.	96-99, 101, 104	Requires support to estimate length, weight, time or volume in terms of standard units *(Sample Activities: 96-99, 101, 104).*	Maybe able to give fairly accurate estimates of length, weight, time or volume in terms of standard units in some situations *(Sample Activities: 96-99, 101, 104).*	Consistently able to give accurate estimates of length, weight, time or volume in terms of standard units *(Sample Activities: 96-99, 101, 104).* Uses estimation meaningfully to take decisions in a number of problem solving situations – in the classroom or elsewhere.
Measurement of complex shapes and use of formulae (7.5)	Computing area and perimeter of shapes, using formulae to compute area and perimeter of rectangles.	106-108	May require support to recall and use the expected formulae to compute area and perimeter *(Sample Activities: 106-108).*	Knows the expected formulae for computing the area and perimeter of shapes. Uses these formulae in routine problems *(Sample Activities: 106-108).*	Knows more than the expected formulae related to the area and perimeter of shapes. Uses these formulae not only in routine problems, but also in problem solving situations in the classroom or in real-life *(Sample Activities: 106-108).*

	Conceptual area	Sample Activities	Proficiency Level		
			Basic	Intermediate	Advanced
	Complex reasoning with area and perimeter.	109-110	Demonstrates an initial understanding of area and perimeter. May require support in attempting complex tasks based on these concepts *(Sample Activities: 109-110).*	Demonstrates a fair amount of clarity in the concept of area and perimeter, but may occasionally get confused between them. Sometimes able to do complex tasks based on these concepts. *(Sample Activities: 109-110).*	Demonstrates complete clarity in the concept of area and perimeter, including distinguishing between the two. Consistently able to do complex tasks based on these concepts *(Sample Activities: 109-110).*
Knowledge and use of instruments for measuring (7.6)	Understanding and using measurement instruments.	111-118	Barely familiar with common measuring instruments – may know it is a measuring instrument, but not what exactly it is used for etc. Requires support to do even straightforward measurement tasks using these instruments *(Sample Activities: 111- 113).*	Somewhat familiar with common measuring instruments – for instance, may know how to use the ruler, but may sometimes require support to use a more complex instrument like the protractor. Mostly able to do straightforward measurement tasks using these instruments *(Sample Activities: 111-113).*	Completely familiar with common measuring instruments like ruler, weighing scales, protractor etc. and knows how to use them correctly in any situation. Consistently able to do straightforward measurement tasks using these instruments, taking care to measure as accurately as possible etc. *(Sample Activities: 111-113).* Able to devise (and articulate) strategies to handle more complex measurement tasks - involving measuring a line/ straight object that is longer than the ruler given, measuring with a 'broken ruler' (with the 0 cm mark missing), measuring angles in any orientation etc *(Sample Activities: 114-118).* Uses these instruments appropriately whenever a situation demands so.©

8

Problem Solving, Patterns and Data Handling

One of the main aims of mathematics education is to help the child develop the right skills and attitude required to solve real life problems. Learning about problem solving, patterns and data handling in the primary classes can equip children in dealing with such problems.

It is necessary for problem solving tasks to be authentic. If a child sees the problem only as something belonging to school, and not something that would interest an adult, she is not likely to be interested in it. On the other hand, an authentic problem, one where children see the benefit of solving it, brings satisfaction and hence enhances problem solving ability. For instance, when I ask a child to distribute 10 toffees among 10 children, the problem is not uninteresting. But it achieves authenticity when I ask further: "How are you sure that you have counted all the toffees and all the children? Is there any way to check?" This is because such verification is a real need for a child, and the ability to do so is immediately seen by the child to be rewarding. It is discussed in more detail in this section. Firstly, it is identifying, visualising and formulating the problem. Secondly, it is devising a strategy to solve it, and thirdly, it is verifying it. The ability to communicate, to support one's solution through argument, to follow an argument are also important facets developed in the problem solving approach.

It is also important to acknowledge that there are problems of varying difficulties. The implication is that from an early age, children should experience attempting problems whose solutions require different durations. Such variety goes a long way towards forming the mathematical predisposition referred to above. Perseverance is an important part of problem solving. We must dispel the myth that any problem that cannot be solved in a short duration is unsolvable.

8.1 Problem Solving

Although most people would agree that mathematics is about solving problems, it is not often clear what is meant by this, especially at the primary level. Mostly, it relates to the so-called "word problems", and in general, end-of-chapter exercises. But clearly there is much more to problem solving, that is linked to the child's skill and understanding of a concept, and to his/her own confidence in formulating and solving problems. Usually problem solving consists of 3 broad steps.

Step 1 **Understanding the problem** – The first step consists of identifying the problem and formulating the problem to be solved is the first important aspect of problem solving. That is, the first thing to look for is whether the child has understood the problem, whether she is able to visualise and represent or model the problem in her own way.

Step 2 **Choosing a strategy** – Next, is the planning of a strategy to approach the problem - to be able to draw on existing knowledge to construct solutions - examine if they are valid, and evaluate possible solutions are all aspects to be assessed. Good problems are those that give children a chance to solidify and extend their knowledge and stimulate new learning. They would need to be meaningful, reasonable and authentic, and not contrived to draw upon some specific operation.

Step 3 **Solving the problem** – Eventually, when children get used to the hints and clues offered, they start anticipating the clues, and that is when they employ heuristics. A heuristic is a plan of attack, which helps in approaching the problem in a particular way. A heuristic is designed to help solvers to approach, understand, and attempt the problem. Checking out if their strategy and solution are valid is also part of this final step in any problem solving exercise.

A range of strategies are usually available for solving problems, such as using diagrams, looking for patterns, or trying special values or cases. These strategies need instructional attention if children are to learn them. Ideally, the curriculum should be structured so as to expose many problem solving strategies to children. Children also need to learn to monitor and adjust the strategies they are using as they solve a problem. Typical strategies appropriate for introduction at various levels may be as follows, with the understanding that they are built cumulatively:

- use manipulatives, "act it out", draw a picture
- guess and check, make a table, list or chart
- look for a pattern, use logical reasoning
- create a model, estimate
- the "work backwards" strategy

During assessment, you need to look for the solution strategy itself and the ability of the child to use multiple skills. The ability to verify and interpret results and to generalise solutions represent a slightly higher stage of problem solving capabilities. Perhaps most importantly, one would look for their attitude towards solving problems, their willingness to preserve, and their ability to approach any kind of problem in a systematic manner.

Typically, problem solving is used to assess childs ability to use knowledge in familiar situations or mere recall the solution of a problem already solved by them previously, or solved by the teacher for them. It is important that real problem solving tasks given for assessment are not merely situations in which the child is 'practising' a procedure learnt earlier – rather the task should give the child full opportunity to use her knowledge and skills in a new situation.

The teacher needs a range of problems for assessment of problem solving ability. Some tasks may be tough problems for some children, exercises for many, and mere recall for others. We must simply remember to challenge all children and provide repeated exposure to problem solving throughout the year. A positive attitude and repeated exposure is critical for success.

Assessment must take into account the difference in problem solving performance between formal "test" situations and informal classroom occasions. The former cause so much stress on the child that her approach may not be what comes naturally to her. Observation is a good assessment tool in the classroom, especially when children are working in groups. A teacher can best assess students' perseverance, self-confidence, interest, cooperation, contribution and much more, in such group environments.

Given below are some Activities that can be used for problem solving. Problem solving is a skill, so the tasks could pertain to any conceptual area in mathematics.

ACTIVITY 119

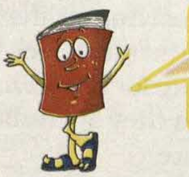
Material: Real material or toys to play shop, toy money.

Instructions: The game is a role play and can be used to motivate children to solve the problem. The teacher or child can act as the storekeeper and give the list of item in the store and price for each items. For example, sugar - Rs 20 per kg, rice - Rs 18 per kg, oil - Rs 50 per litre. The children are asked to act as customers and are given a list of items to be purchased by their parents (for example 5 kilos rice and 1 kilo sugar and 1 litre oil). They have to calculate the total amount of money they have to pay to the storekeepers.

Use the information to make up other problems. For instance:

Your mother gave you Rs 100 to buy 5 kg rice and 1 kg sugar. Is the money enough? How did you figure out?

How many kilos of rice can I buy for Rs 75?



Check the strategies used by the children to count the money, calculate change etc. Check how they decide and whether they can explain when they are asked questions like: "Is the money enough?"

ACTIVITY 120

Material: Paper and pencil.

Instructions: Look at the pattern below and fill in the missing number.

3 ■ 6
5 ■ 10
7 ■ 14
10 ■ ?

What is the missing number? What is the rule?

ACTIVITY 121

Material : Dices with different numbers on the faces.

Instructions: Throw two or more dices and give some instructions related to the numbers that come up. For instance, if the two numbers that have come up are 49 and 50, you could ask the children:

(a) Read the number on the face.
(b) Find the sum.
(c) Suggest two ways of adding the numbers.

Ask them to carry out different operations and do each in at least two different ways.

ACTIVITY 122

Material: Oral task, actual coins may be used is needed.

Instructions: I have six coins worth 6 Rupees and 50 paise. What coins do I have?

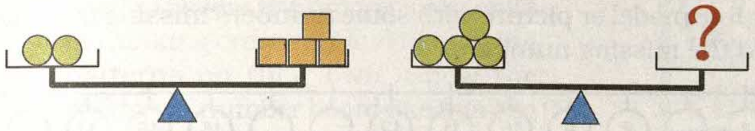
Check out - who is able to come up with a way of doing it? Is there more than one way? If there is someone who is not able to get even one way of making up Rs 6 and 50 paise with 6 coins, see if she can do it by working with actual coins.

ACTIVITY 123

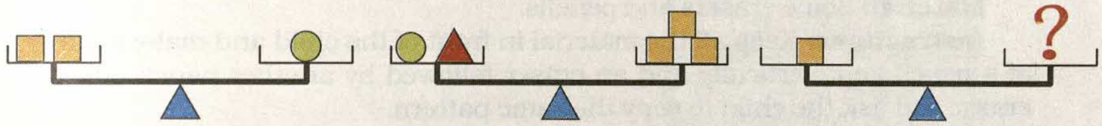
Material: Paper and pencil.

Instructions: Use the concept of balance to develop and assess reasoning ability. Initially, such tasks can be done with a real balance and objects – later it can be modified into paper and pencil tasks involving reasoning.

(a) If 2 balls balance 4 boxes, how many boxes would be needed to balance 4 balls?



(b) Look at the balances below and find out what could be used to balance the thirdscales.



8.2 Patterns

Mathematics curriculum at school level includes the study of patterns. Invariably the curriculum designer desires children to look for patterns and thus enjoy the process of mathematical exploration. This translates to some number patterns and geometric patterns in textbooks, which do seem to interest children.

The development of abilities in the area of patterns include:

- identifying the pattern and developing language for describing a perceived pattern informally
- analysing the pattern to generalise it - i.e. going beyond the observed and verifying it
- engaging with the process sufficiently to cause anticipation and finally, acknowledgment of the need for verification

These could well be listed as exploring, conjecturing, analysing, and proving processes that characterise mathematical thinking.

Several domains are accessible to children for exploration of patterns, beginning with counting. Some of the earliest instances are those of skip counting: 1, 3, 5, 7,... or 3, 6, 9,... etc. Sequences also offer a lot of room for play: 1, 1, 2, 3, 5, 8, ... or 1, 4, 9, 16, 25, ... etc. Some of these can be filled and extended in many ways. 'Primes' are another very rich source, and has also fuelled great mathematics. Pattern explorations of shapes contribute richly to both the formal and informal learning of geometry.

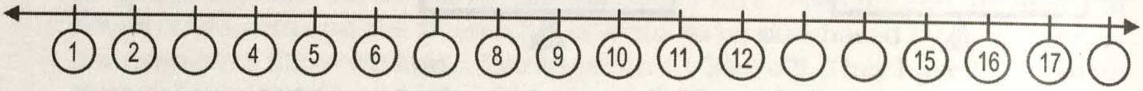
Look carefully at a calendar... it is a rich source of number patterns, and discoveries of relationships. Is there a relation between 3 numbers in a row? In a column? In a diagonal?

MARCH						2008
MON	TUE	WED	THU	FRI	SAT	SUN
					1	2
3	4	5	6	7	8	9
10	11	12	13	14	15	16
17	18	19	20	21	22	23
24	25	26	27	28	29	30
31						

Tasks are usually open ended or providing scope for going from one level to another and interaction with children to elicit their explanations or to alter and elaborate the task slightly to enrich their learning. Patterns could be varied for the complexity of the generative rules. They could also be patterns of numbers or of shapes.

ACTIVITY 124

Material: Number line model or picture with some numbers missing from it.
Instructions: Find the missing numbers.



ACTIVITY 125

Material: Some erasers and pencils.

Instructions: Keep all the material in front of the child and make a pattern of a pencil kept vertically and an eraser followed by another pencil and then eraser and ask the child to copy the same pattern.

The complexity of this task can be increased by using more materials or having a complex pattern. You can also ask the child to describe the pattern in his own words to see how well he can verbalise the relationship.

ACTIVITY 126

Material: Paper and pencils, or worksheet.
Instructions: Observe the numbers and continue the pattern:
3,2,3,2,3,2...
3,3,3,2,3,3,3,2,...
9, 18, 27, 36, 45, 54, 63,...
1,2,1,1,2,3,2,1,1,2,3,4,3,2,1,...
1,2,3,6,12,24,...

In this Activity, parts (a) to (c) are of the initial level but the other two are advanced level tasks requiring the child to identify and analyse the pattern before continuing it.

Also remember, a pattern can be continued in different 'corrent' ways. For instance, one child could continue pattern (a) as 3, 2, 3, 2, 3, 2, 3, 2, 3, 2, ... and another as 3, 2, 3, 2, 3, 2, 0, 3, 2, 3, 2, 3, 2, 0 and soon.

ACTIVITY 127

Material: Paper and pencil
Instructions: Explain the series and then complete it:

(a) 2 → 3
3 → 4
4 → 5
5 → ?
(b) 2 → 5
3 → 7
4 → 9
5 → $\boxed{?}$

Note that part (b) of this Activity is much tougher than part (a) and should be given to children who already are completely comfortable with simpler ones. Do remember to ask the children to explain the pattern in words, and see how clearly and accurately the child is able to do it. For instance, in (b), on being asked to explain the pattern, a child who is very clear might say "If I double the first number and add 1, I get the second, so for 5, I will get 11".

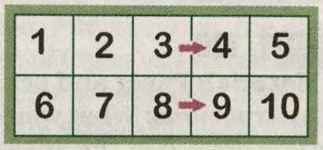
ACTIVITY 128

Material: A board with numbers, or calendar, if the child has her own copy (on paper) of the board used, it is better.

Instructions: This task can be used to assess the progress of a child in thinking creatively and identifying patterns on their own. Show the children a number board like this one (or a calendar) and ask:

1	2	3	4	5
6	7	8	9	10
11	12	13	14	15
16	17	18	19	20
21	22	23	24	25

(a) Do you see any relation between the corresponding numbers in the third column and fourth column?



1	2	3→	4	5
6	7	8→	9	10

(b) Do you see any relation between the numbers in the first row and the corresponding numbers in the second row? And so on.

(c) Find all the number patterns you can in this grid.

The same task can be given with number grids with other number of columns. Children can also be asked to look and locate specific patterns. For instance, "In the above number grid, find the numbers that come in the tables of 4 and colour them".

A variation of the task could be to ask children to make number grids that satisfy some specified conditions. For example:

"Make a grid where the tables of six come in the last column."

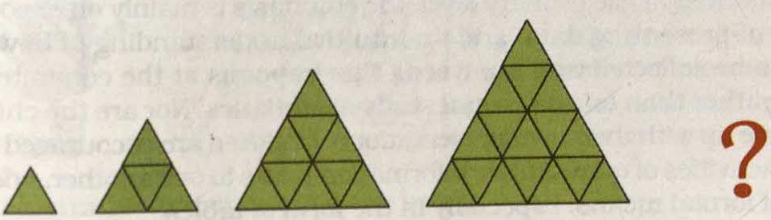
or

"Make a number grid where the tables of 3 appear diagonally."

ACTIVITY 129

Material: Sets of tiles, seeds etc. to create patterns, or patterns drawn on board or paper.

Instructions: Create some pattern, depending on the level of the children using the tiles or material that you have. For instance, if you have triangle shaped like the ones below, you could create a pattern like this.



Now ask the child:

How many triangles do you need to form the next figure? Make the next triangle.

The first question gets children to find the pattern and extend it, but in the second question when you actually ask them to make it, they get to verify it. You may also ask children to do them simultaneously. If some children are found to be doing this task very quickly, they could be asked more difficult questions which require them to generalise the pattern, for instance, "Can you predict how many tiles will be needed to make the sixth figure in the pattern?" See if they can predict by using abstract logic, or whether they necessarily need to do it physically.

Other patterns can be made using tiles, seeds etc or on paper and similar tasks designed.

ACTIVITY 130

Material: Paper and pencil.

Instructions: What would be the tenth item of this number pattern? Explain how you arrived at the answer.

(a) 2, 4, 6, 8, ...
(b) 1, 1, 2, 3, 5, 8, ...

8.3 Data Handling

An ability to collect, classify and interpret data is considered an important aspect of mathematical literacy. Increasingly, use of tables, diagrams and questionnaires is an integral part of everyday life. This is a skill required not only for interpreting any information meaningfully – be it reading newspapers, or even textbook material. Moreover, graphical forms and tables are a critical tool for communication, and enrich the child's toolkit for formal presentation of information. Later on, when the child is called on to critically judge presented data (which is arguably essential for a literate person), one who has not been trained in methods of presentation is likely to be easily manipulated.

Typically data interpretation and analysis is introduced in the upper primary stage and strengthened in the secondary stage of the ten-year school. However, increasingly, in many countries there is a move towards introducing data handling at the primary stage, and this is a view endorsed by the NCF 2005 as well. Important reasons for this include making connections with other branches of study early on (especially language and social studies), as well as relate arithmetic to everyday life situations. A more conceptual reason is that data analysis exposes children to uncertainty and changeability in information, as opposed to the certain and fixed nature of mathematical facts they mostly study.

However, at the primary level, the emphasis is mainly on exposure to various ways of presenting data, and an intuitive understanding of how and why data has to be collected and analysed. This happens at the concretely experienced level rather than as any formal study of statistics. Nor are the children expected to come up with their own presentations. Children are encouraged to comprehend how Activities of quantitative information relate to one another, and learn to make use of formal means, especially in the form of tables.

The following represent the 4 broad areas of data handling which form the basis of children's experiences and which may be assessed:

- **Reading and interpreting data:** This consists of reading data represented in different forms and simple interpretation, beginning with answering questions based on the information in the representation, and understanding what questions can and cannot be answered with the data provided.

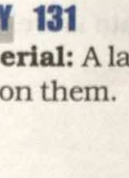
- **Gathering and representing data:** Representing data in the form of tables, pictograms, simple histograms and pie charts, (using suitable units etc.) are all part of this skill.
- **Drawing inferences:** This consists of more complex interpretations and inferences from represented data – where the child has to draw some conclusion based on the data, or verify hypotheses based on the data.
- **Formulating hypotheses and designing an experiment:** This is a more advanced skill, requiring the child to make a hypothesis, understand the nature of the data to be gathered to verify it and later gathering the data and collating it to draw a conclusion. For instance, the child might want to start with a hypothesis: "Children of Class VIII watch less TV than children of Class V" and decide how exactly he would go about checking this hypothesis. Although it seems slightly advanced for primary school children, a task that involves making their own hypothesis and then checking it has the power to help the child see the real purpose of data collection and analysis.

But since the last one is an advanced skill, it could be done more as an skill enhancement activity, rather than as a necessary skill needing assessment.

Thus using information that has already been processed and represented in the form of tables and charts, and beginning with situations and problems which require data to be gathered and represented data to be interpreted are among the natural situations in the classroom that offer themselves for exercises during which assessment of the development of children's capabilities can be undertaken. The subject matter of environmental studies and social studies also offers many opportunities for gathering and handling data.

Note that the objective of such assessment is mainly to see how children prise out relevant quantitative data, how they see the numbers involved possess meaning in context, perform operations on them and apply the result back into the original context to construct new meaning. Moreover, children's understanding of shapes and visual reasoning is called upon in data analysis.

Close attention is to be paid to whether children are overly tied to particular modes of presentation, by presenting the same information in different ways. Moreover, data handling exercises present an opportunity to discriminate whether a child's difficulty is due to linguistic incomprehension, lack of understanding of arithmetical operations, or lack of fluency with numbers.



Data handling is a skill to be developed. So in any of the following Activities, you can formulate further tasks based on any of the 4 key areas of data handling mentioned above, depending on what is the level of the children, what aspect you want to asses, etc.

Some illustrative Activities are given below.

ACTIVITY 131

Material: A large chart divided into 12 columns, with the months of the year written on them.

Instructions: Each child is asked to bring an empty matchbox from home, and to mark it in any way that he/she likes, for identification. (For instance, white paper could be stuck on each match box, with each child drawing a patterns of her choice on the paper). Each child then pastes the match box in the column corresponding to the month of her birth. The teacher can prod the children into thinking about questions like:

In which month do we have the most birthdays?

In which month do we have the least birthdays?

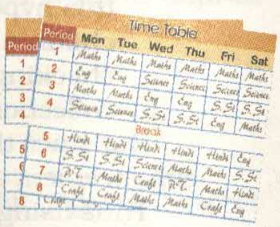
How many birthdays fall in the first three months of the year?

Here children are analysing and interpreting the data and the teacher can assess these skills by asking the above questions to the individual child.

Different tasks like this could be planned, for instance – measuring the diameter of the head of every child etc. Again, if they are working in groups, they can be asked to formulate their own questions and the teacher can assess on the quality of the questions formed.

ACTIVITY 132

Material: Copies of the class time table, or time tables of other classes.



Time Table

Period	Mon	Tue	Wed	Thu	Fri	Sat
1	Maths	Maths	Maths	Maths	Maths	Maths
2	Eng	Eng	Science	Science	Science	Science
3	Maths	Maths	Eng	Eng	S.St	S.St
4	Science	Science	S.St	S.St	Eng	Maths
Break						
5	Hindi	Hindi	Hindi	Hindi	Hindi	Eng
6	S.St	S.St	Science	Maths	Maths	S.St
7	P.T.	Maths	Craft	P.T.	Maths	Hindi
8	Craft	Craft	Maths	Maths	Craft	Eng

Instructions: The class time table has a complex structure, and analysing it opens up ways of unlocking children's understanding. Asking the children to answer the following queries offers much scope to see how well they understand tables:

How many hours per week are spent in mathematics?

Which is the activity that takes up the least time over a week?

Which are the subjects studied daily? On alternate days?

Can play time be doubled, but without reducing time for any subject by more than one hour?

The last question is more advanced than the others. It requires the child to first understand what to do, and then figure out how to analyse accordingly. Note which children can do this independently and who need support for this part of the task.

Similarly, simple tables of bus or train timings of a town can be subject to questioning.

ACTIVITY 133

Material: Material to measure or record as per the task.

Instructions: Collecting statistics (within the classroom) of children's height, weight, favourite colours, foods, games, number of siblings, even length of nose or index finger, can be a source of great entertainment and education, as well as an opportunity for assessment of data handling. In particular, access to information by indexing can be assessed.

How many prefer red to blue?

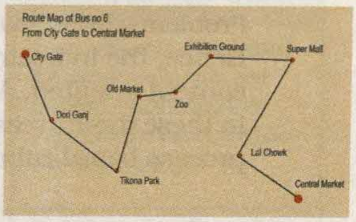
Which is the colour most preferred among girls?

The use of graphics for visual presentation of such data is welcome, mainly to see how much use children can make of such forms.

ACTIVITY 134

Material: Copies of the diagram of a bus or train time table or route.

Instructions: A simple diagram of a bus route (maybe the school bus, if one is used) offers room for questions like:



How many stops does it make?

How many stops between Atif's stop and Arjun's?

Which is the third stop after Asha's stop? etc.

A variation of this task for older children could be to involve them in constructing detailed maps of the school, a road, a village etc. Here, the critical idea is not scale, but relative distances. Distances, even if not absolutely proportional, should be broadly so. It can be designed as a group task where each group draws the map of the same thing or different things. Later, each group could make questions based on their own map (with the questions) and then exchange them with another group, who can try and answer them.

In this task children can be assessed on how accurate their representation is, and also the quality of the questions they formulate. In the questions formulated by them, it should be seen whether the questions can really be answered based on the map.

Later, when they exchange maps, even data interpretation skills can be assessed.

ACTIVITY 135

Material: Material to measure or record as per the task.

Instructions: Before giving this activity the teacher gives the table of height and weight, which shows what should be a child's weight for a particular height. A sample table is shown below. It is only an example and not prescribed by a doctor. A separate table may be needed for boys and girls, since the normal weight ranges for a given height differ for boys and girls.

Height	Ideal weight	Normal weight	Overweight	Underweight
5 feet	50-55 kgs			
5' 1"	56–60 kgs			
etc.	etc.			

The teacher can use the above chart (the ideal weights given are just for illustration).

Ask a child to use the weighing scale and weigh 10 other classmates, and use a height scale on a wall to measure their heights.

Ask the child to classify the children into normal, over weight, under weight.

Observe how the child goes about collecting information - does she use a paper, pencil and a format in which to capture the data?

See if the child is able to classify each child into the categories - observe the approach to approximation, classification.

Observe the child's approach to presenting her findings, and if she is able to communicate the findings well.

8.4 Indicators

Problem solving, patterns and data handling are skills and not conceptual areas. Hence, the indicators for these need to be more **process oriented** rather than content oriented. A sample rubric showing demarcation of skills and attitudes in these three areas is given below. The teacher can modify it or make any other process based rubric of her own.

	Skill area	Sample Activities	Proficiency Level		
			Basic	Intermediate	Advanced
Problem solving (8.1)	Understanding the problem	119-123	Occasionally able to understand the problem on her own. May need support at other times to understand what the problem requires.	Mostly able to understand the problem on her own, but may occasionally need support in the case of complex problems.	Consistently able to understand the problem on her own, and identifies special factors that might influence the solution before choosing a strategy.
	Choosing a strategy	119-123	Occasionally able to choose effective strategies, but may mostly need support from teacher or peers to decide on a strategy.	Sometimes able to choose an effective strategies on her own, but may need support from teacher or peers in the case of new situations or complex problems. May sometimes tend to stick to a few strategies, or stay with one strategy even if it is not workin.	Consistently able to choose effective strategies that help towards solving the problem on her own. Demonstrates awareness of a range of strategies – like drawing pictures, making tables, looking for patterns etc. Shows willingness to periodically revise strategy – does not stay with one strategy for too long even if it is not working.
	Solving the problem	119-123	May try to solve the problem but approach may not be valid, or there may be major conceptual or computation errors in the solution.	ometimes able to solve the problem by using valid approaches with no conceptual or computation errors. At other times, may make computation errors, or demonstrate minor conceptual flaws May need active encouragement to attempt challenging tasks May occasionally need prompting to recheck solution for errors. Explanations or justification may not be fully clear or complete.	Consistently able to solve the problem by using valid approaches with no conceptual or computation errors. Does not give up too easily if a challenging task is presented, rather, enjoys solving challenging problems. After solving the problem, rechecks the solution to check for errors on her own. Consistently able to justify the solution with clear explanations.
Patterns (8.2)	Identifying and generalising patterns	124-130	May require support to recognise patterns – even simple ones. Requires support to articulate the relationship in the pattern or generalize the pattern.	Mostly able to recognise simple patterns, and occasionally even complex ones. Sometimes able to articulate the relationship in the pattern using common or mathematical language, but articulation may not always be clear or fully correct. Able to generalise some simple patterns.	Consistently able to recognise patterns, even complex ones Able to clearly articulate the relationship in the pattern using common or mathematical language. Able to generalise patterns and actively looks for and uses patterns in any problem solving context – whether in day-to-day life, or in the classroom

	Conceptual area	Sample Activities	Proficiency Level		
			Basic	Intermediate	Advanced
Data handling (8.3)	Reading and interpreting data	131-135	Occasionally able to answer questions that require direct reading of data from simple tables, charts, maps etc.	Mostly able to answer questions that require direct reading of data from simple tables, charts, maps etc, may need support to interpret data from more complex charts etc.	Consistently able to answer questions that require direct reading of data – even from tables, charts, maps etc. that are fairly complex for the age level. Able to independently evaluate whether a particular question can be answered on the basis of the given data or not.
	Gathering and representing data	131-135	Mostly not fully attentive and careful while gathering data. May represent the data in the chosen form, but the representation may be inaccurate or not made carefully.	Mostly takes care to gather data carefully. May not be very attentive to aspects like accuracy while gathering data. May represent the data in the chosen form, but the representation may sometimes be inaccurate.	Takes care to gather data carefully. Attentive to aspects like accuracy while gathering data. Understands that inaccurate data can mean inaccurate conclusions or results. Represents data accurately using the chosen form. Able to independently choose an appropriate and effective form of representing the data collected.
	Drawing inferences	131-135	Requires support to analyse the data or draw conclusions based on the data, even in simple cases.	Sometimes able to independently analyse the data and draw conclusions based on the data, may require support in complex cases.	Consistently able to independently analyse the data and draw conclusions based on the data, even in complex cases. Formulates meaningful hypothesis and follows up independently by actually gathering and collating data in order to verify it.

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Bhugeswar Dewri
Niramay Devi
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Hariprasad Misra (H.T.)
Bharati Devi
Bharati Saikia
Minakhi Bordoloi
Bijoy Deka (H.T.)
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Lakhiti Bordoloi
Prabha Saikia
Aysa Siddik
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Smt Minati Hazarika
Rabinda Kakati
Indumati Patowari
Beula Kalita
Pankaj Medhi
Pravawati Deka
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Rupam Hazarika
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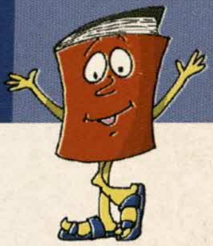
Manorama Medhi
Biju Deka
Bhaben Das
Rosena Begum
Anjali Barman
Rema Begum
Hira Devi
Farida Begum

Notes

Notes



A Time to Reflect

Why Should Children be Assessed?

Since we are all concerned about children's learning, assessment needs to be undertaken for a number of reasons:

- Support and improve every child's learning and development.
- Observe what changes and progress takes place over time.
- Identify individual and special needs and requirements.
- Plan teaching-learning situations in a more suitable way.
- Enhance the child's self-understanding and personal development.
- Achieve curriculum aims/syllabi objectives.
- Improve on-going teaching-learning in classroom.
- Provide evidence of children's progress to communicate to parents and others.

What Should be Assessed?

- Children's learning/performance in different subject areas.
- Achievement of skills – academic, inter personal, etc.
- Interests, attitudes and motivation amongst other aspects.
- Change and progress over time.
- Children's response to educational inputs, situations and/or opportunities.

When Should Assessment be Made?

- Continuously throughout the year.
- Periodic reflection by the teacher 3 or 4 times in a year.

How Should the Assessment Process be Undertaken?

Steps that can be followed are:

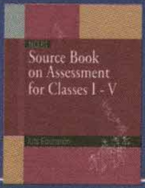
- Collecting information/evidence.
  - A variety of sources
  - Different ways/methods
- Recording of information/evidence.
- Making sense of collected information/evidence.
- Sharing and communicating feedback on assessment.

How can Assessment Information be Used?

It can be used to :

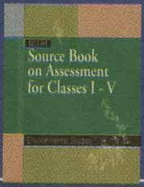
- Improve children's learning and performance.
- Provide the right kind of learning opportunities, materials, aids, equipment etc.
- Bring out the best in children.
- Improve teaching-learning processes.
- Cater to differences in children and their special needs.
- Move children from one level of learning to a higher level.

Source Books on Assessment for Classes I - V



Source Book on Assessment for Classes I - V

Arts Education



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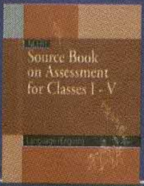

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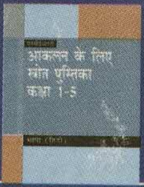
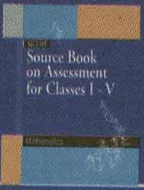
Language - English



आकलन के लिए स्रोत पुस्तिका कक्षा 1-5

भाषा - हिंदी



Source Book on Assessment for Classes I - V

Mathematics

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